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A PARTIAL ANALYSIS OF THE EFFECTS OF TACTUAL-VISUAL STIMULATION BY SPOKEN LANGUAGE.

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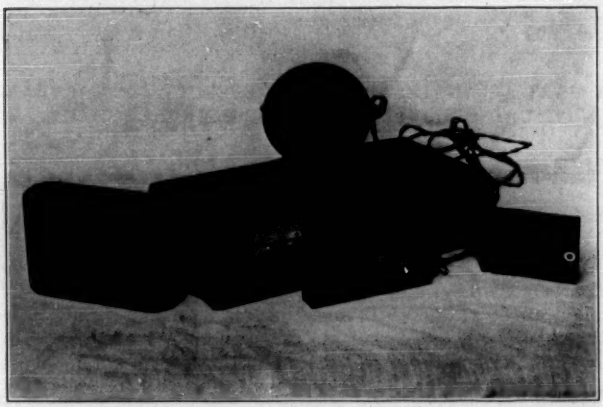
INTRODUCTION—EARLIER REPORTS.

In March 1927, I had the honor to stand here, before members of the Franklin Institute, to discuss what may be called, after a manner of speaking, " 'Hearing' through the fingers."[1] Of course that phrase is taking some liberty with the English language. I am not grafting ears upon fingers nor eyes upon toes. My address dealt with a certain "acquired-in-the-laboratory" function of a remote ancestor of the sense of hearing. What shall I call it: the sense of touch? of pressure? of vibration? Perhaps I am working with more than one sense: with several senses that reside in the skin and possibly in deeper-lying tissues. I was then reporting my actual laboratory experiences up to that time: experiences that had established the possibility of detecting, discriminating and interpreting some forms of speech when the only criteria available were those that were supplied through the sense organs I have referred to. How great a load of language can be carried by the primitive ancestor of the human ear I did not know. And I do not know now. But I have more information on that question of "archæology" now than I had three years ago.

[1] " 'Hearing' through the Sense Organs of Touch and Vibration," JOURNAL FRANKLIN INSTITUTE, Vol. 204, No. 3, Sept. 1927, pp. 329-358.

I described an instrument in 1927—the Teletactor—that has been designed and constructed for my use by Bell Telephone Laboratories for transmitting speech through a microphone and amplifier to a receiver in the fingers of observers or "hearers" who are thus enabled to feel the instrument vibrating in unison with the speaker's voice.



A portable Teletactor: on the left a case containing three small dry cells. Above a microphone containing a mechanical amplifier in its housing. On the right, the receiver: a small post at the center of the white circle vibrates against the skin. Below, a switch.

This instrumentation, then, is the medium through which I have been injecting the human voice into the skin of my subjects or observers or "listeners." In that paper of 1927, I set forth several specific accomplishments of my experiments to that time. First, persons can learn by tactual signs alone to recognize at least ten vowel and diphthongal qualities eighty-three times out of a hundred; they can attain this degree of accuracy in from eight to twenty practise periods of a half hour each. Secondly they have shown their ability to learn to identify certain sentences, 120 in number, by their gross patterns as felt and to attain to from 60 per cent. to complete accuracy in thirty-six practise periods of 40 minutes each. Whether greater or less accuracy had been attained

appeared to depend, in part at least, upon how much prior laboratory experience our subjects had had. Thirdly, I presented data indicating certain pairs of consonants, the members of which can be discriminated tactually. Finally, I reported finding that some tactual stimuli from speech over the Teletactor are more pleasurable than others. All this, I suggested, is quite intriguing in relation to language: something that promotes dreams out of which realities may emerge.

No consideration whatever had been given to the question whether the forms of speech I have mentioned cannot be more successfully received through a lip-reader's eye, for example, than through the skin. We were studying only the receptivity of the organs in the skin preparatory to inquiring into the possibility of training the organism as a whole to the end that this receptivity might be relatively extended, even if not absolutely so. We had only launched a survey: not a program loaded with ambition to develop these sense organs plus mechanical auxiliaries into substitutes for stopped ears. Such a program might not be utterly fool-hardy. But I am staking nothing upon it.

PROGRAM AND ORIENTATION.

What I presented in 1927 may be described as representing, though incompletely, the first stage in a large program of research. It was a partial survey of the capacities of the sense organs I have referred to. It suggested the hypothesis that these sense organs may be enabled to catch forms of speech that cannot be laid hold of by the eye alone observing a speaker's face: and that these organs may serve as auxiliaries to the eye in those cases in which hearing is lacking, a long time before they can possibly serve as substitutes for the ear. I have tested this hypothesis, and 124 records have indicated clearly that simultaneously seeing the face of a speaker and feeling his words over the Teletactor affords an advantage of at least 30 per cent. over "straight" lip-reading. This is the effect of dual stimulation: the simultaneous stimulation of two disparate senses (the sense of vision and the sense of touch [2])

[2] When I use the word "touch" in this paper, it is as a convenient blanket term to cover the cutaneous senses of pressure and tickle and the possibly more problematical deeper sense of vibration.

by the same stimulus source, or speaker. And the general conclusion has been confirmed by the distinguished Professor David Katz, Director of the Psychological Institute in the University of Rostock, Germany, who less than a year ago undertook an experimental examination of my claims on this point.

I am often surprised and even amused when I find it necessary to stop to explain that there is nothing occult or uncanny in the sort of dual stimulation and its effects that I am talking about. The mere mention of a common experience of every one of us should satisfy on that point. We hear our companion's speech and at the same time we see his speaking face. In this situation we interpret more accurately than when we hear but do not see, as when we are separated by telephonic or radio communication. This is a case of dual (visual-auditory) stimulation: stimulation of disparate senses. Vision is made an auxiliary to hearing. Incidentally it becomes more and more an auxiliary with progressive dullness of hearing. I assume that it is unnecessary for me to make such a commonplace observation as this before members of the Franklin Institute for any other purpose than to show how I try to put at rest those whose boot heels have rubbed off their imaginations and who fear that what I am calling "dual stimulation" is so new and unheard of that it cannot fit into a respectable society of ideas.

So the sense of touch may be employed as an auxiliary to the eye in the process of interpreting speech. A detailed account of my experiments in this relation is now on the press and will soon be available.[3] This work, I have already said, may be described as representing the second stage in my program as a whole.

In preparation of this evening's paper, I have been particularly interested in setting out some work that may be described as belonging to a third portion of the entire program. The question is: what are the particular reasons for the advantage of dual (tactual-visual) stimulation? Evidently, at least some characteristics of speech are discriminated more successfully through the skin than through the eye. What are

[3] *Proceedings of the Illinois Academy of Science*, 1929–1930. *Jour. Abnor. Psychol.*, 1930.

they? I am about to present a partial analysis of the question.

Data are accumulating within a fourth division of the program in which we are comparing the performance of the sense of hearing with its analogue, the sense of vibration: such as detection of high frequencies; of differences in intensity; of differences in frequency; and finally we are testing ability to detect the direction of a vibrating body by the organs in the skin and by the ear respectively. Incidental mention of results of some of these comparisons will be made as we proceed.

The foregoing suggests the nature of our entire Laboratory research program:

1. A survey of the receptive capacities of the organs of touch.
2. An examination of the hypothesis that these organs may serve as auxiliaries to the eye of the deaf and hard-of-hearing in relation to the interpretation of speech.
3. A search for whatever detailed characteristics of speech they may be that can be discriminated more successfully by touch than by vision.
4. An experimental study of analogies between the sense of vibration and the sense of hearing.

Before I go on to present the data pertaining to the question in a preceding paragraph and to the title of this paper, I am tempted to spend two or three minutes in an attempt to give this research its setting in the world outside my own Laboratory situation.

As early as 1846, E. Mach described the sense of hearing as an outgrowth from the primitive sense of touch. Indeed, the ear is today sometimes described very crudely as an amazingly sensitive tactual organ.

Dr. David Katz, whom I have already mentioned, has done an important service in bringing together in compact form a survey of the accomplishments of many hands in respect to the senses whose functions we are considering here.[4] In the same connection, he has made us acquainted with his own penetrating researches relating to the senses in question. He expresses his conviction, not only that there is an independent sense of vibration with organs of its own, but that

[4] David Katz, "Der Aufbau der Tastwelt," *Zeitsch. f. Psychol.*, Ergänzungsband 11, Leipzig, 1925, pp. 270.

from the viewpoint of evolutionary development, this sense represents a midway stage between the sense of touch (pressure) and hearing. Its close analogue is the auditory sense and not that of vision. For example, if successive flashes of light stimulate the eye, they fuse and appear to be one continuous brightness when (depending upon their intensity and duration) they occur around twenty-five times a second. But successive stimuli upon the vibratory sense have not fused when, as we have found in our own Laboratory conditions, they are one hundred times as frequent. For the Laboratory has found that these organs easily detect a vibrational stimulus whose rate is 2,600 times a second in certain conditions of intensity: a thousand beyond the highest rate that has ever been published. In other conditions, according to an estimate by Dr. V. O. Knudsen, who is well known among physicists, they may detect a very much higher rate than this.[5] This is of distinct significance, both for theory and practice. Furthermore, however the frequency of successive light stimuli may vary, the resulting sensation quality is not altered. It is otherwise with the sense of vibration. For here the quality changes, according to Dr. Katz, with the change in frequency, analogous to tone in the sphere of hearing.

In addition, Dr. Knudsen, in the course of an extended research, has reported that the cutaneous senses distinguish intensities of stimulation as nicely as the ear detects differences in loudness. They are, on the other hand, much less accurate than the ear in discriminating frequencies or pitches. According to the same authority, two absolute vibrational frequencies must differ by from ten to thirty per cent., depending upon their place in the scale, in order that the difference may be detected. These figures are apparently being somewhat lowered in the course of an investigation by one of my pupils, Mr. W. R. Roberts, that is now going on. If further work should bear out present indications, it will probably be due only to our subjects being more highly practiced than those who coöperated with Dr. Knudsen. Hearing, on the other hand, by far surpasses touch in this relation. It detects differences in pitch when the corre-

[5] V. O. Knudsen, " 'Hearing' by the Sense of Touch," *Jour. Gen. Psychol.*, I, 2, Apr. 1928.

sponding frequencies differ by but a fraction of one per cent.[6] And returning to Dr. Katz: he is authority for the statement that Dr. H. Gutzmann has observed that in the range A to e of the tonal series, one's fingers alone enable one to distinguish vibrational stimuli when their actual difference corresponds to one whole tone.[7] Professor Knight Dunlap also has published some data on the same question and has reported that differences as low as 5 per cent. in the absolute frequency of forks can be detected by the organs in the skin.[8] We anticipate that before long additional experiments now in progress will cast some further light upon the parallelism between hearing and the sense of vibration. Suffice it to say here, without adducing further proof that, in the words of the late and greatly lamented Dr. Victor C. Vaughan in the course of a comment upon our experiments: "We hear not only with our ears, but with our whole bodies."

What we have so far pointed out gives the work I have in hand a place alongside of physical science and phonetics for the reason that we are transmitting speech instrumentally, and we must therefore be in contact with physical problems that are peculiar to that process. At the same time our problems fall within the range of the biologist. I have already referred to the hypothesis that the sense of vibration is a half-way station between the sense of touch (pressure) and that of hearing. And contemporary researches on hearing in fishes in the Zoölogical Institute at Munich, for example, under the hand of Professor K. v. Frisch are suggestive of the same hypothesis. In a recent publication Professor v. Frisch has reported that one of his laboratory fishes, having had its ears isolated by operation from the central nervous system, made the same discriminatory reactions to tonal frequencies that it had made before.

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[6] *Op. cit.*

[7] H. Gutzmann, "Ueber die Bedeutung des Vibrationsgefühle für die Stimmbildung Taubstummer und Schwerhöriger. Verhandl."

b. "Ueber die Grundlagen der Behandlung von Stimmstörungen mit harmonischer vibration," *Archiv. f. Laryngol. W. Rhinol.*, 31.

c. "Untersuchungen über die Grenzen der Sprachliche Perzeption," *Zeitschr. f. klin. Medizin*, **60**, 1907. (See Katz, *op. cit.*, p. 196.)

[8] Knight Dunlap, "Palmesthetic Difference Sensibility for Rate," *Am. Jour. of Physiol.*, **29**, Nov. 1911, pp. 108–114.

However, I interpret this work as pertaining mainly to psychology: this for the reason that I am interested not only in the absolute receptivity of the organs in question, but especially in the possibility of training the human organism as a whole, to the end that these receptors may practically extend their function so far as to become reactors to stimuli to which they have hitherto been unused.

It is quite fascinating, moreover, especially in this after-the-war period, to conceive of the whole enterprise as a study of the ability of an individual to get on in spite of a handicap. As a matter of fact that is just what the research amounts to. Can a runner adjust himself to the need for running when his feet are both in one and the same bag or will he fall helpless in the attempt?

It is encouraging to recall that up to the present moment, at least, no subject or observer in our Laboratory has suffered the fate of the adjustable chameleon that was described by an open-eyed gardener sitting in his kitchen corner. "The beast," he said, "became green when he was placed upon a green surface, red upon red, yellow upon yellow. But when he was set upon a piece of Scotch plaid, he just 'natchally' died trying to make good."

ANALYSIS OF EFFECTS OF DUAL STIMULATION.

And now at last I return to our demonstrated effects of dual (tactual-visual) stimulation from spoken language. What are the precise sources of its advantage? Why is one more successful in the interpretation of speech when one simultaneously feels a speaker's words and sees his face than when one sees the face but does not feel the vibrations that correspond to the spoken words? Evidently, as I have already said, in our Laboratory situation where the Teletactor is employed, at least some characteristics of spoken language are more readily detected and distinguished through the fingers than through the eye.

HOMOPHENOUS WORDS AND CONSONANTAL QUALITIES.

I have already published convincing evidence that this advantage may arise in part in relation to "homophenous" words;[9] that is words that look alike on the face of a speaker.

[9] "On the Discrimination of Homophenous Words by Tactual Signs," *Jour. Gen. Psychol.*, II, 2 and 3, Apr.–July, 1929.

"Bead," "peat" and "meat" are examples. The conformation of the speaker's face does not vary among these words, and visual signs alone do not enable one to distinguish them. But they feel very unlike through the Teletactor system, owing in part, at least, to differences in intensity of consonantal qualities; to their duration and to slight differences in pattern as between M and N, for instance, and between the semi-vowels L and R. Among 102 groups of such words, I found only eight in which discriminations could not be made very successfully by touch though much less so, or not at all, by vision.

It is quite probable that in our particular Laboratory tests on the effects of dual stimulation the sense organs in the skin served as an auxiliary to the eye exclusively because of just such felt indicators as I have mentioned. The statement has especial weight in relation to our experiments in which the test material consisted not of spoken sentences, but of words uttered singly and not as parts of sentences. For in all such instances, the words (200 in number) were monosyllabic. Of course there are felt differences among vowel and diphthongal as well as consonantal qualities—and differences that are prominent enough to enable one to learn them by feel exclusively through the Teletactor. This was shown in my former paper before this Institute. It is quite improbable, however, that feel has a sufficient advantage over vision, in relation to vowels and diphthongs, to make it an auxiliary to the eye.

With further reference to the question of felt differences among consonantal qualities: one of my students, Mr. Milton Weichbrodt, has made definite progress at finding how far each of several subjects can go toward discriminating every consonantal quality (including semi-vowels) following long *e* —eeb, *e.g.*—from every other consonant quality. They are taken in pairs and in series of twenty. The subjects knew in advance what pair was being employed at a given time and their responsibility was to report which of two they felt first. Both the sense of touch alone and the sense of vision alone (the subjects seeing the face of the speaker in the act of speech but not feeling his words) are being requisitioned to discriminate members of successive pairs of stimuli. Thus the experimenter is able to compare the efficiency of the two senses in

this relation. The experiment, as a whole, requires that each one of several subjects shall make a grand total of 2,400 judgments of discrimination by way of each sense.

The work has progressed far enough with one of the subjects (a novice as a lip-reader and almost equally so in relation to the Teletactor, as far as this problem is concerned) to supply a good index to the final product. This subject has now gone through with 120 pairs of stimuli by each method of stimulation. In relation to 35 of them, the finger has made a better record than the eye.[10] It has been surpassed by the eye in 63 cases, and in 22 there is no advantage either way. In reference to the 35 cases referred to, the general per cent. of error on the Teletactor was 5; in the same cases the per cent. of lip-reading errors was 33. I offer this as incomplete but significant evidence of its sort. In itself alone it is sufficient, without recourse to additional experimentation, to predispose favorably even the uninitiated to the proposition that touch and vision combined should made a better record of interpretation than either one alone can do. And I submit that here is partial explanation of the fact that it is so.

LOCALIZATION OF ACCENT.

But are there not situations in which there are other than consonantal and vowel characteristics of speech that can be transmitted more advantageously to the senses of touch and vibration than to the sense of vision? I am thinking now of conditions in which polysyllabic words and sentences are employed. But first we will consider words alone and later sentences. Here accent is a carrier of meaning and if experiment should prove that its location can be detected by the sense of touch alone more successfully than by vision alone, we should discover one more point at which this sense is capable of making a contribution in the dual stimulation situation.

Experiments upon this point, however, bear upon the general question of the advantage of the dual stimulation and not

[10] The 35 pairs in question are as follows: een-eeng, eek-eer, eeg-eek eek-eel, eed-een, een-eeg, eeng-eex, eek-eez, een-eel, een-eex, eed-eet, eet-een, eet-eel, eep-eeb, eek-eev, eed-een, eer-eex, ees-eez, eet-eeng, eed-eef, eeg-eel, eek-eeng, eev-eef, een-eez, eet-eeg, eed-eel, een-eek, eeb-eem, eez-eex, eep-eem, eeg-eeng, eed-eeng, eeb-eet, eed-eek, ees-eex.

upon the particular situation in which we demonstrated the the superority of *dual* as compared with *single* stimulation. For in this situation 94 per cent of more than 1000 words comprised in 116 sentences that were used as stimuli were monosyllabic and every one of the 200 stimulus words used alone—*i.e.* not as parts of sentences—was monosyllabic also.

Our first task in this relation was to discover how far touch can be relied upon as a means of localizing accent in bi-syllables altogether irrespective of the question whether the same work can be done more effectively by the sense of vision.

Accordingly we made up a list of thirteen nonsense syllables, no two of them containing the same vowel quality. Every syllable begins with *L* and ends with *S*.[11] The list of bi-syllabic compounds is made up by combining every nonsense monosyllable with every other: "lees-laws" for example. In the course of the whole experiment, there were 26 distinct utterances of which "lees" is a part and 26 of which every other syllable is a part. In 13 instances it is a first syllable and in the same number it is a second syllable. The subject had ten opportunities at each utterance to locate the accent, or 260 opportunities at every compound of which *lees* is a part, and 260 at every compound of which every other syllable is a portion. This makes a grand total of 3,380 opportunities to judge whether the accent was placed on the first or the second syllable. Furthermore, because the experiment was performed in full in three situations (that is with three types of receiver) the subject made a grand total of 10,140 judgments. He made a score of 91.8 per cent. correct judgments based upon 3,380 judgments when he was using a single unit receiver with a diaphragm whose natural frequency is 900 d.v.; 87.9 per cent. when he was using a single unit receiver whose moving part is not a diaphragm, but a reed with a natural frequency of 1,600 d.v.; 87.9 per cent. when the receiver was a multiple unit instrument: one unit applied to the thumb and one to each finger of the left hand. Each of the five moving parts in this instrument is a reed whose natural frequency is 1,600 d.v. This instrument is operated through a filter so

[11] The list of syllables is as follows: lās, lăs, lēēs, lōs, lo͞os, lo͝os, lers, lahs, laws, līs, lĭs, lŭs and lars.

that each finger is stimulated by a certain group of frequencies and no other.

The highest score was made by means of the single unit instrument with a diaphragm. Whatever device was in use, the greatest difficulty was encountered when the accent fell upon lees, lŭs and lĭs, and the difficulty is undoubtedly due to the low transmissibility of these syllables over the particular instruments that are available. These figures probably represent with fair accuracy the utmost that can be expected of one's fingers in a situation like ours—given such instruments as ours for transmitting the energy of speech, and given, at the same time, subjects equal to ours in point of experience and interest in this sort of work. I should say in this connection that ours was a hearing subject whose auditory sense was insulated by recourse to our usual sound proof box as a container for the receiver, by ear plugs, by a sound barrier or neutralizer, and by interposing distance and closed doors between him and the speaker. It is possible that deaf subjects might have made a better record than he, due to their greater accustomedness to the use of tactual and vibratory sensation. But on the other hand, anything gained on this score might have been offset by a lack of interest and purposiveness. So I repeat that in the accomplishment of our subject in relation to the localization of accent, we have a measure of the extent to which the capacity of the fingers alone measures up to this sort of task.

Our next question is this: how successfully can the sense organs in the fingers compete with the eye in the art of localizing accented syllables? This is the real question. For if by any means fingers should surpass eyes in this respect, we have found one more point at which they may serve as auxiliaries to the eye in the dual stimulation situation. On the other hand, if eyes alone can surpass fingers alone, the data we have already presented forthwith become much less interesting than they otherwise would be.

The question calls for a new procedure. Deaf lip-reading subjects must be substituted for those who hear. Furthermore, the stimuli we selected for this portion of our work were by-syllabic nonsense compounds in which the syllables were duplicated: "loos-loos" and "dub-dub." In this manner,

we were able to guarantee that, but for differences due to accent alone, one syllable in the compound would carry over our Teletactor System as perfectly as the other. I have already drawn attention to the fact that in a compound such as "lees-loos," accent of the first syllable may be obscured in the Teletactor by reason of the fact that the vowel $\overline{oo}$, *e.g.*, actuates the instrument more energetically than does the vowel *ee*. The other stimulus word "dub-dub" was selected because it is easily spoken and contains in each syllable a short vowel that is both preceded and followed by a consonant and that makes a short sharp impression upon the finger.

SUBJECTS OR OBSERVERS.

The subjects I have already intimated were deaf lip-readers as follows:

F., 30 years old; residual hearing, right 10 per cent., left 20 per cent.; lip-reading ability fair. She has been reading lips for more than 20 years.

T., A successful business man, 60 years old; deaf only two years; residual hearing 0; lip-reading ability very poor.

B., A young woman 25 years old; residual hearing, right 0, left 22 per cent.; lip-reading ability excellent; deafened at the age of six.

D., A young woman 25 years old; residual hearing, right 0, left 50 per cent.; lip-reading ability excellent, deafened at the age of ten.

EXPERIMENTERS.

Three experimenters participated in this work. They are designated as G., C., and D. The first two have been intimately associated with the work from the beginning. D., on the other hand, was introduced as a control. He was entirely in the dark as to the purpose of the particular experiments I am about to report until after he had made his contribution. Why it was desirable to introduce such a control experimenter will be made clear as we proceed.

METHOD.

No subject was given preliminary drill at locating accent either by lip-reading, or by touch through the Teletactor. Experimenter G. made sure that each subject, in his turn,

had a conception of *accent*, as stress or duration or pitch. No detailed explanation was offered to the effect that any one of these characteristics may designate an accent; any two of them, or that all three may be combined in an accented syllable. Accent was illustrated by the expedient of gesturing visibly by hand and head while pronouncing a word. Subjects T., B., and D. were thrown into this work on the occasion of their very first appearance in the Laboratory—within ten minutes after they had taken their seats. F., on the other hand, before she began to participate in the experiments, had already spent seven half-hour periods in the Laboratory. During these periods, she had been occupied at learning to recognize the tactual patterns of a few colloquial sentences in a preliminary effort at improving her speech.

The stimuli (loos′-loos and loos-loos′; dub′-dub and dub-dub′, whichever nonsense word was being used in a given session) were arranged in short series of twenty each, so that in unknown order, the accent would occur ten times on the first syllable and ten times on the second. One short series was given for lip-reading; the experimenter's face, of course, being fully exposed and uniformly lighted. The faces of subject and experimenter were uniformly six feet apart. The subject responded only by the word "first" or "second," according as she thought the accent had been placed on one syllable or the other. The experimenter made the record. Following the presentation, the same stimuli (in a new order of accents) were presented to the subject's cutaneous organs in the ball of her thumb through the Teletactor. In every touch series the experimenter held the microphone, not before his mouth, but against his larynx. In this series, the experimenter's face was hidden behind a screen. In session after session, the lip-reading series was given alternately first and second. In each session, each short series was given the same number of times as the other: varying from three to five.

What has been said to this point regarding method of procedure relates fully only to the situations in which experimenters G. and C. were participating as pronouncers of stimuli. Obviously the arrangement made it possible for each experimenter automatically and unconsciously to give the subject a signal. Assuming that in a given case, the

subject begins hesitatingly to report "first" or "second," to indicate which syllable has just now been accented, the experimenter, observing her face, could give her a cue by unwittingly moving his head to express approval or the opposite.

Experimenter D. was imported to get us away from this possible source of error. It has already been stated that he was ignorant of the purpose of the experiment. He knew before he came in and during the course of his participation only that he was to pronounce "loos-loos" time after time, placing the accent here or there, according to direction from a second experimenter, G., who took his position behind the subject and out of her sight. D. participated only in relation to subject F. Throughout the experiment, he was blindfolded so he could not see the subject. He, therefore, could not give her a cue as G. and C. may conceivably have done, when they, pronouncing the stimuli, had the subject's face in clear vision. Experimenter G., behind the subject, as already described, gave D. the oral signal "first" or "second," whereupon D. pronounced "loos-loos," placing the accent accordingly. The subject responded by raising one or two fingers to indicate where she thought the accent had fallen. G. made the record. No others were in the room. When the first short series had been presented (it was for lip-reading), G. proposed a minute's rest, in the course of which, without D.'s knowledge, a screen was inserted between D. and F. and the subject took the receiver of the Teletactor System into her fingers. The next twenty presentations were for touch: not for lip-reading. At the end of this short series, during a minute's rest the screen was removed and the receiver was laid face down in the subject's lap, out of her hand, again without D.'s knowledge. *In every short series, whether for lip-reading or for touch, the microphone was against D.'s Larynx and the Teletactor was active.* This detail of method was introduced in order that D. might not be disturbed by hearing and then not hearing the receiver and in order that he might not find a cue therein that might enable him to jump to the conclusion that he was speaking now for lip-reading and now for something else. This is a situation in which it is highly desirable for what I call an experimenter to be saturated with highly distilled ignorance.

RESULTS.

The per cents. of successful localizations are tabulated below:

TABLE I.

Showing the Per Cent. of Successful Localizations of Accent by Lip-reading Alone and by Touch Alone Respectively.

Stimulus: Loos'-loos, Loos-loos'

Subject.	Speaker.	No. Reactions to Each Mode of Accent.	Lip-reading.		Touch.	
			Right	%	Right	%
F.	G.	240	135	56.2	174	72.5
F.	C.	230	158	68.7	192	83.5
F.	D.*	100	49	49.0	83	83.0
T.†	G.	50	5	10.0	37	74.0
B.	G.	100	76	76.0	94	94.0
D.	G.	100	78	78.0	90	90.0
Stimulus: Dub'-dub, Dub-dub'						
F.	G.	50	29	58.0	40	80
B.	G.	50	40	80.0	50	100
B.	C.	50	42	84.0	47	94
D.	G.	50	35	70.0	40	80
D.	C.	50	34	68.0	44	88

* Control Experimenter. See above "Experimenters."
† No lip-reading ability. See above "Subjects."

The total number of reactions to "loos-loos" was 820, and to "dub-dub" 250 for each mode of accent. In respect to the first compound, the per cent. of correct reactions averages 56.3 in the lip-reading series and it mounts to 82.1 in the touch series. Corresponding figures relating to the second compound are 72 and 88.4. Averages in cases of this sort are of little consequence. But inspection of the detailed figures in the table reveals striking gains in the touch series. And a separation of successful reactions to accent on the first syllable from such reactions to accent on the second syllable indicates that when subjects were lip-reading and accent was on the first syllable, they reported correctly in 61.2 per cent. of cases; when they were lip-reading and accent was on the second, they were correct in 61.7 per cent. of cases. When they were relying solely upon touch, and accent was on the

first. they were right in 81.4 per cent. of instances and in 81.9 per cent. when accent was on the second syllable.

When subjects were relying upon tactual stimulation alone in relation to "dub-dub" and the accent was upon the first syllable, they were right in 94.5 per cent. of cases. When it was upon the second, they were right in 83.1 per cent. of instances. When they were depending upon lip-reading and accent was on the first syllable, they were right in 40 per cent. and on the second, in 76 per cent. of cases.

Both in the lip-reading and the touch series, in respect to stimulus "loos-loos," there was an advantage of 0.5 per cent. in favor of accent on the second syllable. In respect to "dub-dub," judgments from tactual cues were considerably less accurate when accent was on the second than when it was on the first syllable. In the lip-reading series, the case was sharply reversed.

ELEMENTS OF ACCENT.

Of course the foregoing is not a complete analysis of the situation respecting any of our stimulus words. Accent is made up of at least three elements: stress or intensity, a temporal factor and pitch. The isolation of these elements calls for kymographic or oscillographic records of accented contrasted with unaccented syllables. In our situation it was impracticable to make such records as these simultaneously with the presentation of the stimuli to our subjects. They were of necessity made subsequently to the main portion of the experiment that I have already discussed. As between the presentation of stimuli to our subjects, therefore, for their discrimination and later presentation to the mechanical recording device, the conditions are altered so radically that we are confronted by a new and distinct investigation that is now on the way.

CONCLUSION.

To this point, it can be unqualifiedly asserted as a fact that with respect to our stimuli, feel is superior to vision as means of locating accent. There can be no question that in every instance in which we are dealing with equivalent components of compound words, these results will hold. Furthermore, I repeat that our results of 3,380 attempts at locating accents in nonsense bi-syllabic compounds when the syllables incorporate

thirteen different vowel qualities with uniform consonantal elements show that we may expect localization by touch to occur in approximately 91 per cent. of cases.

LOCALIZATION OF EMPHASIS.

Now the finger in contact with the Teletactor, having proven its superiority to the eye in the art of localizing accented syllables, beckens us to consider the possibility of its making a contribution of a similar nature but much more important. What about the localization of emphasis within a sentence? Here is a more important indicator of meaning than the placing of accent within a word can possibly be. Let me say to you:

"It would be a GREAT mistake to do that";
and again:
"It WOULD be a great mistake to do that";
once more let me say:
"It would be a great MISTAKE to do that";
and finally:
"It would be a great mistake to do THAT."

In each instance I have used the same words as in every other. But have I said the same thing four times over? Every reader of English knows that by dint of placing my emphasis here or there, I have expressed four different meanings in these sayings. Consequently, missing the emphasis, one may grasp the words and let the meaning slip unless by chance context or gesture or both are there to give assistance.

Is it the finger on the Teletactor or the eye alone that affords the more reliable information respecting the location of emphasis in these sentences?

I have sought to answer these questions by experiment upon two deaf young women, L. and C., aged 20 and 22. They had had no Laboratory training prior to this experiment. In their right ears they had a residuum of hearing amounting to 10 and 15 per cent. respectively and somewhat less in the left. Both had been deafened at approximately the age of eight. As lip-readers both were rated as "very good" on the basis of their reaction to the tests that were employed in the course of a survey of schools for the deaf by the National Research Council in 1924.

The four sentence stimuli above were arranged in 40 short series of ten each to be presented to the subjects' fingers alone and the same number to be presented to them as lip-readers without contact with the Teletactor. When a short series of ten had been presented for lip-reading, it was followed by another short series through the Teletactor and that by a lip-reading series and so on alternately until each subject had made 400 reactions as a lip-reader and the same number of reactions to the Teletactor.

The experimenter in this case is designated by M. He was unacquainted with the purpose of the experiment. He was blindfolded throughout. In all details the modus operandi was practically a copy of that already described in relation to the control experiment on the localization of accented syllables. A second experimenter, G., standing behind the subjects and out of their sight, gave M. the cue and he pronounced the stimulus accordingly.

RESULTS.

Each subject's finger surpassed his eye as an agent for localizing emphasis in the stimulus sentences quoted above as the figures in the following table indicate:

TABLE II.

Per cent. of Success in Localizing the Emphasized Word by the Finger Alone and by the Eye Alone.

400 Trials Each Way.

Word.	Subject L.		Subject C.	
	Eye, %.	Finger, %.	Eye, %.	Finger, %.
Would	72	87	74	88
Great	68	85	71	83
Mistake	67	82	72	80
That	70	79	70	75

We have no indication as to whether emphasis is detected by dint of stress, duration, pitch or all combined. The determination of this question in detail, like the same query in relation to accent, can best be treated as a separate research problem.

SUMMARY.

Finally but a few words by way of summary of this research enterprise that it has been my privilege to initiate, so far at least as it relates to spoken language, and to carry along with more or less intensity during several years to its present state.

During more than a year after it had got well under way, the investigation was simply a survey of the capacity of certain sense organs in the skin and deeper tissues to receive and to discriminate forms of speech such as vowel qualities, isolated words and sentence patterns, without regard to the question whether the same forms can be discriminated more successfully by the eye observing the face of the speaker. The answer to the questions implied in the survey was affirmative. The spoken forms I have mentioned can be discriminated with sufficient clearness through the aid of a suitable instrument, to enable one to learn to identify sentences, words, and some of their elements. This indicates that there are tactual patterns corresponding to the forms that were employed in the experiments and that these patterns possess a degree of stability. Were this not so, learning could not occur.

In the next place, the products of the survey played a part in projecting the hypothesis that the feel of speech may be made an auxiliary to vision and so assist the deaf in the interpretation of speech. In the course of testing this hypothesis: comparing the accuracy of interpretation by the process of lip-reading alone (visual stimulation) with the accuracy attained by the process of lip-touch-reading (simultaneous visual and tactual stimulation), we have accumulated 124 records by which the hypothesis in question is established. The advantage of dual (tactual-visual) over single (visual) stimulation is at least 30 per cent. This conclusion, as I have said, has been confirmed by Dr. Katz of the University of Rostock.

This has opened up the third question, a partial answer to which has been supplied in this paper: what are the particular points at which the advantage of dual stimulation has arisen? The tactual sense is superior to the visual in respect to the discrimination of members of certain groups of homophenous words: 94 groups out of 102.

An inspection of the detailed results that support the last statement led us to a direct examination of the feel of the impact of consonant qualities. To this date, almost 5,000 attempted discriminations by means of visual signs and by tactual criteria have brought to the front 30 discriminations out of a possible 80 that are made more successfully tactually than visually. A single discrimination of two consonants, it may be observed, that the organs of touch can make, but that is impossible by means of vision, may be the critical factor that gives one an insight into the meaning of a word or of a whole sentence.

The accent of a syllable, furthermore, often assists in conveying the meaning of a word—as hearing people know well. This characteristic can be detected and its location described far more accurately by aid of the sense of touch alone than by vision alone. The advantage of the former is represented by scores approximately 20 per cent. higher on the average than those that pertain to vision. Approximately the same situation holds with respect to the location of the emphasized word in a sentence, which, incidentally, is more significant for meaning than is accent within a polysyllabic word.

Other characteristics of speech—elements of pattern in spoken discourse, one may say—are yet to be studied by a method similar to that that has been described in this paper.

For example: is the finger or the eye superior in detecting the longer and shorter pauses that abound in conventional speech, and the alterations in tempo that are likewise more or less conventionalized in our spoken discourse? All such characteristics as I have mentioned here, when taken together, constitute the crude pattern of speech that is meaningful in itself, as every intelligent observer discovers to his sorrow when he passes from his native land to another with less familiar spoken language.

In the fourth place, this course of research provokes many questions that are of extraordinary scientific interest because they extend to comparisons between certain capacities of the sense of touch and those of the ear. What distinctions of rate and of intensity and what discriminations of direction of the source of vibration can be made by touch and by hearing

respectively? At least tentative answers to the first two questions are now available. Depending upon the range of frequency in question, ears are probably ten to sixty times as refined as touch in respect to pitches, but only on a plane of equality with it in relation to discriminations of intensities. Work now in progress may refine upon these answers and illuminate or darken the hypothesis I have mentioned earlier—that the sense of vibration is a way station on the road toward the emergence of the sense of hearing. But I am not under the illusion that all this is less than the work of a lifetime.

And finally, the work we have already done and the extraordinary capacities for discrimination in the organs in the skin and deeper tissues that have already been demonstrated suggest the feasibility of a considerable extension of the uses of the senses in question in relation to the training and education of the deaf, and to their assistance as a group in the audience-room, or individually in home or office. This suggestion, as far as it relates to formal education, has now for a year and a half been proving its fertility in a school for the deaf in Chicago, both in respect to the reception of spoken language and to the development of speech in mutes and in semi-mutes.

INTEGRAPH SOLUTIONS OF ELECTRON ORBITS IN THE BARKHAUSEN-KURZ EFFECT.

BY

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INTRODUCTION.

Barkhausen and Kurz,[1] in 1920, found that a triode having cylindrical elements, when operated with a positive grid and a plate at filament potential or somewhat more negative, could become a source of extremely high frequency oscillations. The period of the oscillations was not affected by the constants of the external circuit, but was determined by the spacing of the tube elements and the grid and plate voltages. The following explanation of the effect was proposed: an electron emitted from the filament is drawn toward the positive grid, and, after passing through it, is repelled by the plate, passes through the grid again, and continues to oscillate in this way within the filament-plate space of the tube until it strikes one of the tube elements.

Since 1920, Gill and Morell,[2] Scheibe,[3] Kapzov and Gwosdower,[4] Hollmann,[5] Knipping,[6] and many others have carefully studied the effect. While other explanations of its origin have been proposed, it will be assumed here that the original theory of Barkhausen and Kurz is correct.

The fundamental period of the oscillations is assumed to be the time required for an electron emitted from the filament to complete an excursion into the grid-plate space and return to the filament. For a tube having cylindrical elements,

[1] Barkhausen and Kurz, *Phys. Zeitschr.*, XXI, 1920, p. 1.
[2] Gill and Morrell, *Phil. Mag.*, (6), **44**, 1922, p. 162.
[3] Scheibe, A., *Annalen d. Physik*, **73**, 4, 1924, p. 54.
[4] Kapzov and Gwosdower, *Zeitschr. f. Phys.*, **45**, 1927, p. 114.
[5] Hollmann, H. E., *Annalen d. Physik.*, **86**, 1928, p. 129.
[6] Knipping, P., *Zeitsch. f. Hochfrequenztechnik*, **34**, 1929, p. 1.

this time has been computed by Scheibe,[8] who finds the following expression for the wave-length:

$$(1)\quad \lambda = \frac{4cr_1}{\sqrt{2\frac{e}{m}V_{fg}\,10^8}}\left\{f\left(\sqrt{ln\frac{r_2}{r_0}}\right) + g\left(\sqrt{\frac{V_{fg}}{V_{fg}-V_{fp}}ln\frac{r_2}{r_1}}\right)\right\}.$$

Where

$$f(x) = x\epsilon^{-x^2}\int_0^x \epsilon^{u^2}du$$

$$g(x) = x\epsilon^{x^2}\int_0^x \epsilon^{-u^2}du.$$

r_0, r_1, r_2, are the radii of filament, grid, and plate respectively. V_{fg} and V_{fp} are the absolute values of the grid and plate potentials with respect to the filament. The other letters have their usual significance. The wave-lengths as computed from this equation agree well with observed values, both as to dependence on tube dimensions and on grid and plate potentials. No attempt is made, however, to account for the fact that observable electrical oscillations can be produced, for although the motion of any electron emitted from the filament might be oscillatory about the grid, a certain "coherency" of the motion is essential, if any oscillations in an external circuit are to result.

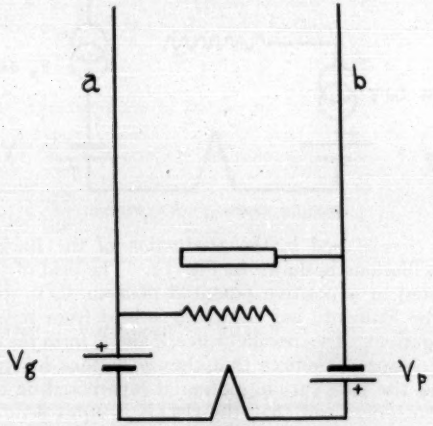
In a recent paper, Kapzov[7] has assumed that the oscillating cloud of electrons gives rise to an alternating potential superposed on the steady grid and plate potentials, and shows that in a field of this type only those electrons emitted from the filament during a limited portion of a cycle will ever oscillate about the grid, and the remainder will either move directly to the plate, or will be drawn back into the filament. (This of course applies only to those electrons which pass through the grid without striking one of the grid elements.) In this way, a group of coherently oscillating electrons may be formed in the tube, which would account for the effects observed.

The equations of motion for an electron in such a field

[7] Kapzov, N., *Zeitschr. f. Physik*, **49**, 1928, p. 395.

cannot be solved in terms of known functions, and the results obtained by Kapzov are derived by making use of a number of "comparison" electrons, which are assumed to leave the filament at various phase angles (of the superposed alternating potential) and to move under the influence of a steady field.

FIG. 1a.



Circuit used in production of Barkhausen-Kurz oscillations.

In this way, he finds the limiting phase angles of emission between which oscillatory motion may result, and later solves the equations by step-by-step integration for a few cases, tracing the motion for about one cycle (if oscillatory) or until the electron strikes either the plate or the filament. His results are derived only for the special case in which the plate potential is zero, and the effects of space charge and of initial velocity of emission are neglected.

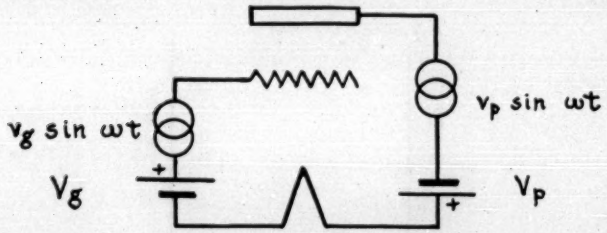
Now the equations which enter in this problem are particularly adapted to solution by the Continuous Integraph recently developed at the Massachusetts Institute of Technology by Dr. V. Bush,[8] and it was thought worth while to

[8] Bush, Gage, and Stewart, JOURNAL FRANKLIN INSTITUTE, Jan. 1927, pp. 63–84.

Bush and Hazen, JOURNAL FRANKLIN INSTITUTE, Nov. 1927, pp. 575–615.

trace through a number of cases and compare the results with those obtained by Kapzov, as well as to take into account the effect of various factors which were neglected in his work.

FIG. 1*b*.



Equivalent circuit when tube is oscillating.

The circuit used in the production of the Barkhausen-Kurz oscillations is shown in Fig. 1*a*. The grid of the tube is operated at a positive potential of from 50 to 350 volts above the filament, and the plate is held from zero to 100 volts negative. Two parallel wires, *a* and *b*, form the external circuit. Kapzov assumes that the oscillations set up in the tube give rise to a varying potential superposed on that due to the batteries, and that the circuit is equivalent to that shown in Fig. 1*b*, where alternating voltages of magnitude $v_g \sin \omega t$ and $v_p \sin \omega t$ are introduced into the grid and plate circuits respectively. Under these conditions the potential difference between filament and grid is

$$V_{fg} = V_g + v_g \sin \omega t, \tag{2}$$

while between grid and plate it is given by

$$V_{gp} = V_g + V_p + v_g \sin \omega t + v_p \sin \omega t. \tag{3}$$

If we assume $v_g = v_p = v$,

$$\begin{aligned} V_{fg} &= V_g + v \sin \omega t, \\ V_{gp} &= V_g + V_p + 2v \sin \omega t. \end{aligned} \tag{4}$$

Then the equations of motion of an electron in the filament-grid space and grid-plate space become respectively,

$$(5)\qquad \frac{d^2r}{dt^2} = \frac{k_1}{r}(1 + \alpha \sin \omega t),$$

$$\frac{d^2r}{dt^2} = -\frac{k_2}{r}[(1 + \gamma) + \beta \sin \omega t],$$

where

$$k_1 = \frac{e/m\ V_g}{ln\ r_1/r_0}, \qquad k_2 = \frac{e/m\ V_g}{ln\ r_2/r_1},$$

$$\alpha = \frac{V_g}{v}, \qquad \beta = \frac{2v}{V_g + V_p}, \qquad \gamma = \frac{V_p}{V_g}.$$

These equations are of the form

$$(6)\qquad \frac{d^2r}{dt^2} = f_1(r)g_1(t) \quad \text{filament-grid space},$$

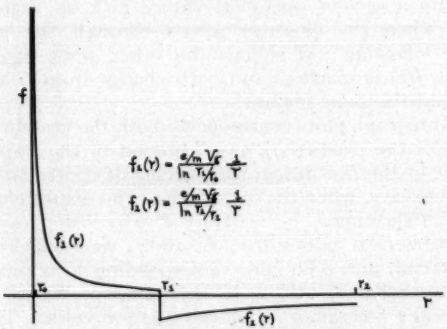
$$\frac{d^2r}{dt^2} = f_2(r)g_2(t) \quad \text{grid-plate space},$$

where

$$(7)\quad f_1(r) = \frac{k_1}{r}, \qquad f_2(r) = -\frac{k_2}{r},$$

$$g_1(t) = 1 + \alpha \sin \omega t, \qquad g_2(t) = (1 + \gamma) + \beta \sin \omega t.$$

FIG. 2.

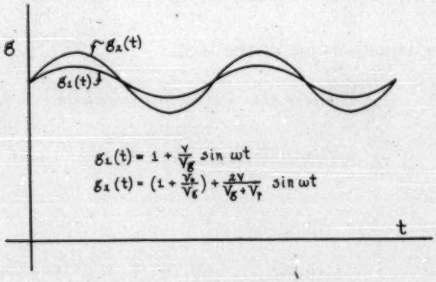


Plot of $f(r)$ against r as set up for integraph solution.

To set up the equations for solution on the Integraph, the functions $f_1(r)$ and $f_2(r)$, Fig. (2), are plotted on one sheet,

and the functions $g_1(t)$ and $g_2(t)$, Fig. 3 on another sheet. The plot of $g(t)$ against t is mounted on one table of the Integraph and driven from right to left at constant speed, the value of $g(t)$ being followed with a pointer by one operator. The other table carrying the plot of $f(r)$ against r, is driven by the result of the second integration, so that its displacement at any instant is equal to r, and $f(r)$ is followed with a

FIG. 3.



Plot of $g(t)$ against t as set up for integraph solution.

pointer by a second operator. Since $g(t)$, as well as $f(r)$, changes when the electron passes through the grid, the operator following $f(r)$ signals the other when the discontinuity in $f(r)$ is reached, and both change from f_1 and g_1 to f_2 and g_2 at the same instant.

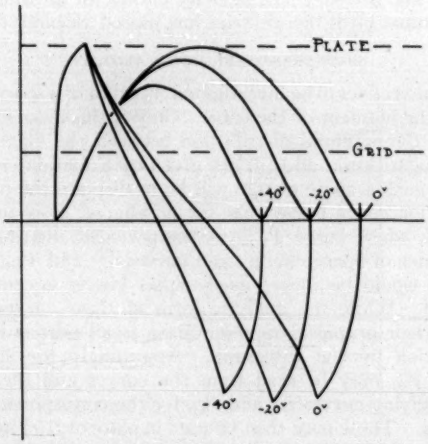
The Integraph plots continuously both the first and second integrals of the product fg as a function of the upper limit, the result of the first integration giving of course the instantaneous velocity, while the second gives the displacement as a function of the time.

The numerical values of r_0, r_1, and r_2 were taken as 0.006 cm., 0.31 cm., and 0.80 cm., corresponding to a tube investigated by Scheibe, and were the same as those used by Kapzov. V_g was taken for all cases as 100 volts. The amplitude of the alternating component must be assumed, as there is no way of computing its value. This was assumed by Kapzov to be 10 volts, and accordingly the same value was used in the first solutions, although later on the effect of

altering this quantity was investigated. For the special case where V_p is zero, the form of the two plots is shown below.

The frequency of the alternating component, ω, is assumed to be the same as that of the periodic electron oscillation in the absence of an alternating field. This will be referred to as the "natural" frequency. Its value may be computed from equation (5), putting $\alpha = \beta = 0$, which then reduces to equation (1) of Scheibe. Solutions of this equation were found for the three values of plate potential $V_p = 0$,

FIG. 4.



Velocity and displacement curves as a function of the time for plate potentials of 0, −20 volts, and −40 volts. Grid potential +100 volts, and no alternating field.

$V_p = -20^v$ and $V_p = -40^v$, the grid potential being $+100^v$ in all cases. The same three problems were then set up for solution on the Integraph, and the resulting curves used for calibration. These Integraph solutions for the displacement as a function of the time are given in Fig. 4, together with the corresponding velocity curves. The computed wave-lengths for the three cases are 154 cm., 125 cm., and 107 cm. The corresponding distances on the Integraph solution are 42.0 cm., 35.0 cm., and 28.5 cm. The ratios are

$$\frac{154}{42.0} = 3.68, \qquad \frac{125}{35.0} = 3.58, \qquad \frac{107}{28.5} = 3.75.$$

$$\text{mean} = 3.67.$$

i.e., 1 cm. on the Integraph solution corresponds to a distance of 3.67 cm., or a time of 1.22×10^{-10} seconds. This fixes the scale of t.

It will be noted that the curves in Fig. 4 are similar in form, the effect of a negative plate voltage being to reduce the distance of penetration into the grid-plate space, and to decrease the period. The velocity curves for all three orbits are the same until the electron has passed through the grid.

SPACE CHARGE AND INITIAL VELOCITY.

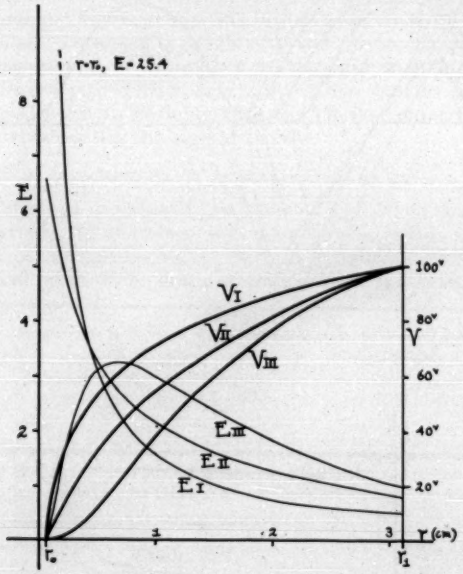
The next effect to be investigated was that of a space charge around the filament of the tube. Only if the space charge is zero will the potential distribution between the filament and grid be logarithmic, and $f_1(r)$ be given by Equation (7). The effect of such a space charge will be to displace the potential distribution curve below that for no charge, as is suggested in Fig. 5, where curve V_I gives the potential distribution in the absence of space charge, and curves V_{II} and V_{III} suggest how this would be altered as the space charge is continually increased. While the analytic form of these curves is not known, their graphical representation is all that is required for solution by the Integraph. Accordingly the slopes of V_{II} and V_{III} may be read from the curves and the results plotted, giving curves E_{II} and E_{III} for the corresponding field strengths. These may then be used in place of $f_1(r)$ in Fig. 2.

Some orbits corresponding to these cases, in the absence as yet of an alternating field, are shown in Fig. 6. Curve 1 is the same as the curve for $V_p = 0$ in Fig. 4. Curve 2 shows the effect of assuming a space charge corresponding to V_{II}, other factors being unaltered. The period is somewhat increased, the general characteristics of the motion, however, being the same.

Curve V_{III}, Fig. 5, has been drawn for the special case where the space charge is such as to reduce the field at the filament surface to zero, so that were it not for their initial velocities of emission, no electrons would leave the filament.

If there is no difference of potential between the filament and plate, however, these electrons will all reach the plate with their original velocities and hence cannot oscillate. Accord-

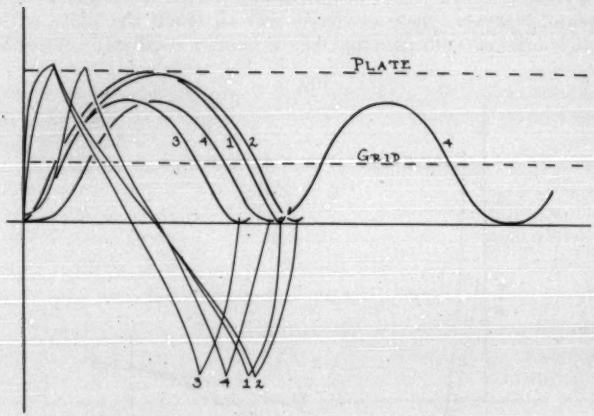
FIG. 5.



Potential distribution between filament and grid (heavy curves), and corresponding field strength in arbitrary units (light curves), as these may be affected by space charge.

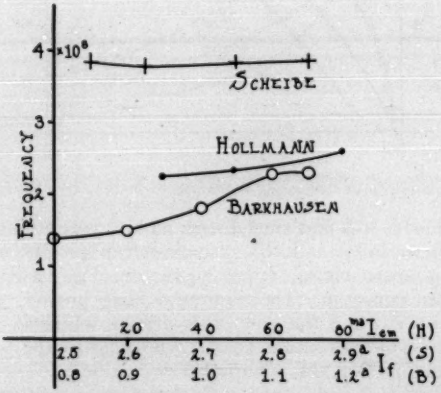
ingly an orbit was first traced with a plate potential of -20^{v}, assuming an initial velocity of emission of 4 volts, and using the same space charge curve as before, V_{II}. This orbit is shown in curve 3. The potential distribution was then altered to V_{III} and the new orbit drawn, which is shown in curve 4. Curves 1 and 2, then, correspond to the same conditions except that the potential distribution has been altered from V_I to V_{II}, and 3 and 4 to the same conditions except that V_{II} has been changed to V_{III}. The conclusion from these

FIG. 6.



Velocity and displacement curves showing increase in period as space charge is increased.

FIG. 7.



Observed frequency of Barkhausen-Kurz oscillations as a function of filament current and emission current.

curves is that the effect of increasing space charge is definitely to increase the periodic time, or to decrease the frequency, a result not in accord with experimental results, which however are meagre. Some results of Barkhausen, Scheibe, and Hollman are plotted in Fig. 7, in which frequency is plotted against filament current or emission current, both of which should correspond roughly to the space charge effect. The trend is for the frequency to increase with increasing space charge or to remain constant, rather than to decrease. This would seem to indicate that the effect cannot be completely explained by the simple theory.

INTRODUCTION OF AN ALTERNATING FIELD.

The effect of superposing an alternating field on the steady field due to the batteries was next investigated, i.e., the solution of Equations (5). With a potential distribution corresponding to V_I a choice of constants of the Integraph is

FIG. 8.

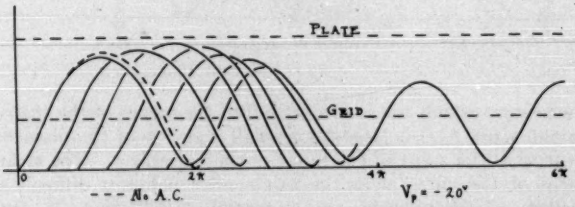
Orbits originating at various phase angles of the alternating component. Plate potential zero.

necessary which tends to exaggerate the errors of the instrument, since $f_1(r)$ is large for a small region near the filament, but small for most of the range of the variable r. The steepness of the curve near the origin also makes it difficult to follow. Consequently the potential distribution V_{II} was used in the remainder of the solutions.

Three series of orbits, originating at various phase angles, were determined for plate potentials of 0^v, -20^v, and -40^v. These three sets of curves are shown in Figs. 8, 9, and 10. The dashed curve in Fig. 8 is the orbit in the absence of an alternating field, the timing curve for the alternating com-

ponent. The full line curves are orbits originating at the phase angles noted, under the influence of a superposed field of the "natural" frequency. Except for the change from V_I to V_{II}, the conditions are the same as those assumed by Kapzov, two of whose curves are given for comparison. The conclusions of Kapzov are borne out, namely, that oscillatory motion is possible only for electrons leaving the filament within limited portions of the cycle. The 0° electron makes its first oscillation in about the same time as in the absence of an alternating field, but the distance penetrated into the grid-plate space is less. This distance increases for successive orbits up to about 90°, and the 90° electron strikes the plate as shown. Similarly all orbits up to about 210° terminate on the plate. The 240° electron does not strike the plate, but does return to the filament after a single excursion, while the motion of the 300° electron is again oscillatory. It will be seen that all electrons emitted at phase angles between 90° and 270° will be unable to enter into any more or less permanent oscillatory group, and that continued oscillatory motion is possible only for such as leave the filament during the remainder of the cycle. These "cut-off" angles agree substantially with those found by Kapzov, which were 70° and 290° respectively.

FIG. 9.

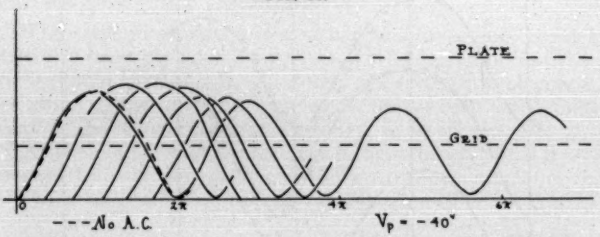


Orbits originating at various phase angles of the alternating component. Plate potential −20 volts.

In Fig. 9 are shown the orbits obtained when a negative potential of 20^v is applied to the plate. None of the paths terminate on the plate, but nevertheless about the same degree of coherency will result as in the previous case due to the termination of orbits on the filament, the limiting angles

lying between 60° and 120° and one side, and 180° and 240° on the other. Increasing the plate potential to -40^{v} results in the curves of Fig. 10. The frequency increases, the amplitude of the motion is cut down, but the degree of coherency is about the same. The formation of a stable oscillatory group of electrons is hence possible, even if the negative plate potential is sufficient to prevent the termination of any paths by the plate.

FIG. 10.



Orbits originating at various phase angles of the alternating component. Plate potential −40 volts.

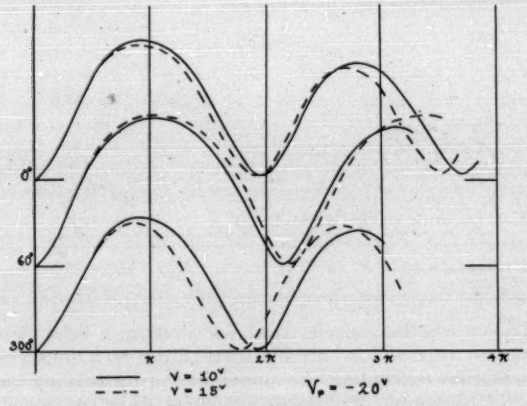
To see whether an electron, not striking a tube element on its first trip, will eventually settle down to a steady state of oscillation (excluding the possibility of its striking one of the grid elements), a number of orbits were continued for several complete cycles, of which only one, that of the 0° electron, is shown in Figs. 8, 9, and 10. While the motion seems to become quite regular, the period is considerably shorter than that of the natural oscillation. Probably the phase relations will eventually become such that the electron is thrown out of the oscillating group.

AMPLITUDE OF ALTERNATING COMPONENT.

As was stated earlier, the amplitude of the alternating component must be assumed, and this was taken throughout the preceding work to be 10 volts. Some of the orbits were redrawn, taking this value to be 15 volts, to see if the resulting frequency were more nearly in accord with the natural frequency. Three of these orbits are shown in Fig. 11, in which the 0°, 60°, and 300° orbits of Fig. 9 are compared with the

same orbits when a larger alternating amplitude is assumed. The full lines are the orbits when $v = 10^v$, and the dashed lines are those found for $v = 15^v$. It will be seen that increasing the value of v increases the period for some orbits and decreases it for others. (The vertical lines represent phase angles of the natural frequency.) The effect seems to

FIG. 11.



Orbits showing effect of increasing amplitude of the alternating component.

be to increase the departure from the natural frequency, orbits of longer period (60°), being increased still further, and those of shorter period (0° and 300°), decreased. It would be of interest to see if a smaller value of v would produce the opposite effect, and tend to make all orbits coincide more nearly with the natural frequency.

SUMMARY.

The Continuous Integraph has been used to trace the orbits of an electron oscillating within the filament-plate space of a three-electrode vacuum tube, such oscillations being presumably the origin of the Barkhausen-Kurz Effect. The effects of space-charge, initial velocity of emission, and amplitude of oscillations are considered. The results check and extend those of Kapzov found by other methods.

ULTRA SHORT RADIO WAVES.

BY

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INTRODUCTORY.

The term short waves is a rather indefinite one, and has generally been considered to mean wave-lengths of less than one hundred meters. It is a purely relative term, of course, and when we say ultra short waves, we use an even less definite term, of the second order of relativity, so to speak.

At the Hague Conference held in September, 1929, it was decided by international agreement to adopt officially the following nomenclature for the classification of wave-lengths.

Long	3000 meters up
Medium	200–3000 meters
Intermediate	50– 200 meters
Short	10– 50 meters
Ultra short	less than 10 meters.

The scope of this paper is confined to the ultra short wave band, and deals mainly with wave-lengths of the order of one meter or less.

EARLY EXPERIMENTS.

The production of very short electromagnetic waves is not at all a recent development. In Hertz's original experiments in 1888 he employed one oscillator which operated on a wave-length of 60 centimeters.[1] This particular oscillator consisted of two copper sheets each 40 centimeters square and 60 centimeters apart connected to two polished gilt balls two or three centimeters apart. Spark coil excitation was used to set the system in oscillation, and the wave-length determined by calculation from the mechanical dimensions of the circuit.

Lodge in 1890 obtained oscillations in a single metallic sphere five centimeters in diameter, but found it much easier to obtain oscillations with larger spheres.[2] Sir J. J. Thompson

has shown that the oscillations from pole to pole of a charge upon a conducting sphere causes radiation of wave-length equal to 1.4 times the diameter of the sphere. Thus in the case of Lodge's five centimeter sphere the wave-length was seven centimeters.

Nichols and Tear[3] in 1923 published the results of some work in which a method of obtaining electrical oscillations was developed by which wave-lengths down to 1.8 millimeters were produced. In this system the oscillator consisted of two tiny tungsten cylinders which formed the electrodes of an oil-immersed spark gap. In the particular oscillator with which the 1.8 millimeter waves were obtained, these cylinders were 0.2 millimeter long and 0.2 millimeter in diameter, separated by a gap of 0.01 millimeter. The system was excited by applying a potential of 30,000 volts at frequencies of 500 to 1000 cycles to the oscillator, a water resistance, and a condenser, all in series. A radiometer type of receiver was used.

The longest heat waves so far measured have a wave-length of about 0.3 millimeter. Thus there remained to be explored a gap extending from 0.3 to 1.8 millimeters, to complete the spectrum between heat and electromagnetic waves.

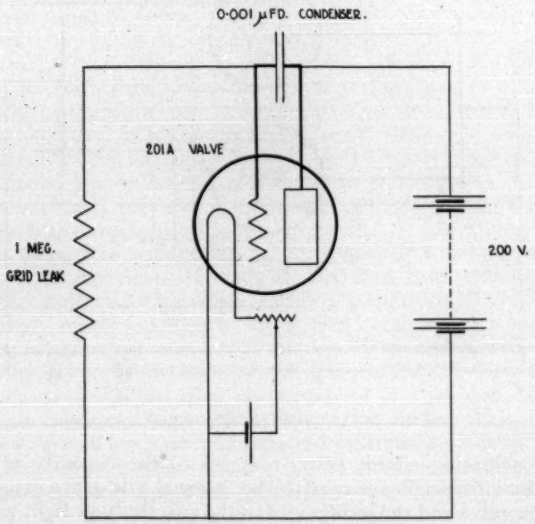
Within the last few years experiments carried out by a number of investigators have resulted in the closing of this gap, and over a narrow overlapping band of wave-lengths it is now possible to produce oscillations by either heat or electrical methods. This work has been mainly done by extending the lower limit of electric waves, as it is found that above about 0.3 millimeter in wave-length oscillations are more easily produced by electrical than by thermal methods.[4] Glagolewa-Arkadiewa has obtained waves down to 0.082 millimeter in length by electrical means.[5] She used as oscillators finely divided metallic particles suspended in a viscous oil. A coating of the resulting gummy mixture was applied to the surface of a wheel rotating between the sparking electrodes.

TYPES OF VALVE OSCILLATORS.

Turning now to electron tube oscillators we find that investigations have been carried out along a number of lines with a view to producing electrical oscillations of shortest possible wave-length.

It has been found by a number of experimenters that stable oscillations may be obtained down to about two meters in wave-length using standard types of vacuum tubes and simply reducing the constants in conventional types of oscillating circuits. Below about two meters major difficulties begin to appear. High power tubes can no longer be used because the dimensions of the valve elements themselves are

FIG. 1.

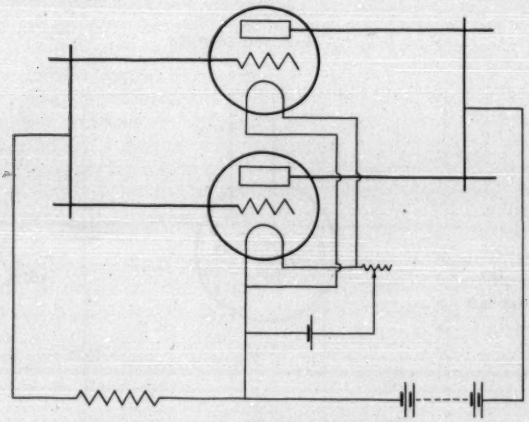


ULTRAUDION OSCILLATOR.

sufficiently great, even though the external oscillating circuit be reduced to the smallest possible mechanical dimensions, to maintain the natural wave-length of the circuit at some value possibly in the neighborhood of three or four meters. One is thus compelled to use low power valves, usually either five or seven and a half watt transmitting valves, or else receiving tubes.

By removing the four prong base of a standard 201a type vacuum tube and connecting a mall 0.001 microfarad fixed condenser between the grid and plate leads as close as possible to the glass envelope, a short wave oscillator is obtained which will give wave-lengths down to below one and one-half meters (Fig. 1). This circuit is really the familiar ultraudion circuit,

FIG. 2.



MESNY ULTRA SHORT WAVE CIRCUIT.

the oscillating circuit being reduced to the capacity of the fixed condenser in series with the internal grid-plate capacity of the valve and the inductance of the two short straight wires, each possibly an inch long, making connections with the grid and plate.

In an oscillator of this type the oscillations are not very stable, and the wave-length is not variable since the tuning of the circuit is fixed at a value determined largely by the internal interelectrode capacity of the valve. By extending the length of the grid and plate wires and moving the fixed condenser up and down these wires the wave-length may be varied and adjusted, but this immediately increases its length to two meters or more.

The push-pull circuit developed by Mesny has been widely used in experimental work, and produces somewhat more stable oscillations than the above circuit as well as allowing greater control at shorter wave-lengths Fig. 2). This circuit uses two vacuum tubes whose grids and plates are paralleled through inductances, the center tap of each of which is connected to the filament. The grid and plate inductances usually take the form of parallel straight wires joined by a shorting bridge. With this circuit stable oscillations may be obtained down to below one meter, and the wave-length can be varied by sliding the bridges along the grid and plate wires.

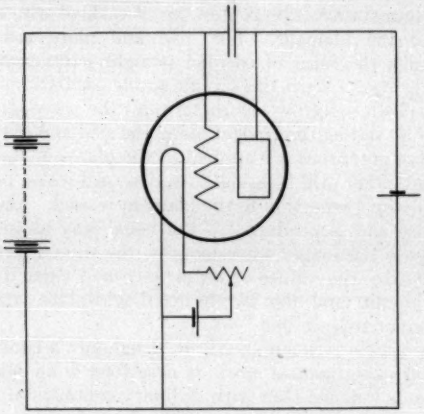
Positive potentials are applied to the plates in these oscillators, and either grid biassing batteries, grid leaks, or simply a straight wire connection to the filament is used. Oscillation takes place and is produced in the usual way as in circuits operating on the longer wave-lengths, the wave-length being determined by the values of inductance and capacity in the external circuit, and also by the fixed grid-plate capacity of the particular tube in use.

In an article written by C. R. Englund[6] a considerable amount of experimental work is described from which the conclusion is reached that with ordinary commercial vacuum tubes now on the market the lower limit attainable is a wave-length of about one and a half meters. This agrees with the investigations carried out by the author for the Research Council of Canada during the summers of 1926 and 1927. In these experiments an oscillator was obtained working on a wave-length of less than one hundred centimeters, but by the time the wave-length is reduced to a value as short as this, the oscillating circuit has been reduced to the smallest possible mechanical dimensions, almost no variation or control of frequency is possible, and the oscillations are sometimes none too stable. The types of circuits used were mostly more or less conventional circuits with the inductances and capacities reduced to very small values. The oscillating circuit inductance usually degenerated into straight wire leads joining a small capacity to the grids and plates by the shortest possible route.

A distinctly different type of high frequency oscillator was developed by Barkhausen and Kurz about 1920.[7] The

usual plate and grid potentials were reversed in polarity, a high positive voltage being applied to the grid and a small negative or zero voltage being applied to the plate (Fig. 3). It

FIG. 3.



CIRCUIT FOR THE PRODUCTION OF BARKHAUSEN-KURZ AND GILL-MORRELL OSCILLATIONS.

was found that this connection could be used to produce extremely high frequencies, wave-lengths of the order of one-half meter and less being obtained. The explanation was that the oscillations took place entirely within the tube, and consisted of pure electron field vibrations about the grid. The hot filament emitted clouds of electrons in the usual way, and these electrons were drawn at a high velocity towards the positively charged grid. A large proportion pass through the grid spaces, are repelled by the negative anode, again attracted through the grid, and repeat the process. Thus the frequency of oscillation is largely determined by the time required for the individual electrons to traverse the interelectrode spaces. This in turn depends on the applied field, so that the frequency

is considerably affected by the potentials used. No external oscillating circuit is used and the constants of the external circuit do not affect the wave-length generated. The amount of energy radiated by an oscillator of this type is, as might be expected, a very small quantity and can scarcely be considered to be of value except for measuring purposes.

In experimenting with the Barkhausen-Kurz type of oscillations two other investigators, Gill and Morrell, found a third type of oscillation produced.[8] Using a circuit of the Barkhausen variety (Fig. 3), having positive grid potentials and negative plate potentials, oscillations were obtained having the properties of the usual type of circuit in which regeneration builds up oscillations in a resonant circuit. In the Gill-Morrell experiments the values of inductance and capacity in the external circuit control the frequency. Wave-lengths obtained are considerably lower than those obtained with the first type of circuit in which positive plate potentials are used.

Another system which has been investigated for the production of very high frequency oscillations uses the magnetron vacuum tube. It is found that a two element vacuum tube may be made to generate oscillations under certain circumstances. If the anode consists of a circular cylinder with the cathode a long straight wire filament at its center, the superimposition of an electromagnetic field of uniform strength with its direction parallel to that of the filament, will, if the field is strong enough, prevent electrons from the filament from reaching the plate, and they will move in circular orbits whose diameter is less than that of the plate. It is found, however, that there is sometimes a small radio frequency current flowing to the plate which may be detected on hot wire meters. Japanese investigators have used this fact to obtain some promising developments in the production of very short wave-lengths.[9] A minimum wave-length of 5.6 centimeters was obtained. To get the shortest possible waves the anode diameter had to be reduced and this decreased the intensity of the oscillations. It was found that the wave-length could be calculated approximately from the semi-theoretical formula $\lambda = 2ct$, where c is the velocity of light and t is the time taken for an electron to travel across the anode-

cathode space. A receiver of the Barkhausen type was employed.

FREQUENCY DETERMINATION.

When a circuit has been obtained which gives stable oscillations at high frequency, the problem still remains to find out exactly what this frequency is. In the case of the oscillators of Hertz, Lodge, Nichols and Tear, and other extremely high frequency oscillators of that type, the frequency was determined by calculation from the measured constants of the circuit. This can be done with whatever type of oscillator is used, but it is usually desirable to have some convenient method of accurately determining wave-lengths by comparison with some calibrated variable standard. The usual method of loosely coupling a calibrated circuit, consisting of a known value of inductance and capacity to the oscillating circuit, can be more or less satisfactorily used down to about two meters. The indication of resonance is obtained either from a thermocouple or hot wire instrument in the wavemeter circuit, or from the deflection of the plate current meter in the oscillator circuit as the two circuits come into resonance. However, the mechanical dimensions of the capacity and inductance of this type of wavemeter become very small when one gets down to about two meters. Using a General Radio Company vernier condenser having five semi-circular plates about one and a half inches in diameter, maximum capacity 15 micromicrofarads, to obtain a wave-length of two meters would require an inductance of a single turn coil approximately one inch in diameter. It is obvious that to produce any measurable indication of resonance with an oscillatory circuit of similar dimensions, quite tight coupling must be employed between wavemeter and oscillator. This, of course, introduces errors into the readings, and it can readily be seen that the constants necessary to obtain a wavemeter of this type to measure waves 100 centimeters or less in length, would require impracticably small values of inductance and capacity. By far the most satisfactory method of measuring these short waves is to use the arrangement devised by Lecher. A pair of parallel wires is coupled to the oscillator and standing waves produced. A shorting bridge on these Lecher wires is slid backwards and forwards and indicates the position of the

nodes and antinodes of the standing waves. The wave-length is thus determined by direct measurement of the distances between these points. This type of wavemeter is not as simple as it may sound, if very accurate results are desired, but a number of refinements can be introduced. The indications of nodal points may be obtained either by observing the deflection of the plate current meter in the oscillator, or by using an indicating device directly in the shorting bridge on the Lecher wires. If the latter method is used, a small correction must be introduced due to the effect of the thermocouple, neon tube, or other indicating device used in the bridge. Wave-lengths down to a few centimeters in length may be readily measured with the Lecher wires type of wavemeter.

POWER OBTAINABLE.

In the oscillators of Hertz, Lodge, and Nichols and Tear, the amount of power radiated is extremely small. Very delicate receivers can give measurable indications of reception up to distances of only a few meters. With what we shall call the normal type of valve oscillating circuits, that is, those in which oscillations are built up at the natural period of the circuit by regenerative action, the maximum power available is of the order of a fraction of a watt, since to produce waves as low as 100 centimeters, the smallest type of receiving tubes must be used.

With Barkhausen-Kurz types of oscillators the amount of power obtained has, so far, been very small, since low power valves have been used in the experiments. It would seem that greater power might be obtained by the use of high power valves in a circuit of this nature. The Gill-Morrell type of oscillations have been obtained with considerable intensity, as compared with Barkhausen-Kurz oscillations, but the maximum amount of power involved is still a matter of but a few watts. Here again there appears to be room for investigation with high power vacuum tubes.

The magnetron oscillator was originally developed on a high power scale, tubes having been designed to operate at power frequencies with an output of many kilowatts. As the frequency is increased, the dimensions of the tube must be decreased, so that less power can be used. However, at a

wave-length of 40 centimeters, sufficient oscillating energy can still be produced (probably a few watts) to enable readable signals to be heard at a maximum distance of one kilometer.[10] In this particular experiment the actual wave-length employed was 41 centimeters and the anode voltage 1,000, with 900 cycle modulation. Single Hertzian resonators were employed at both transmitter and receiver, with the addition of a parabolic reflector and collector at the transmitter and receiver respectively. A system of director chains and wave canals was also employed with a very marked improvement in results. These consist of a series of Hertzian resonators, that is, simply full or half wave vertical antennæ, mounted along the axis of the parabola of the reflector. The receiver consisted of a crystal detector at the center of the Hertzian resonator, with a three-stage amplifier for the modulation frequency. A Barkhausen type of receiver has also been employed with somewhat better results in detecting modulated waves in the neighborhood of 1.5 meters.

Modern investigators have been able to obtain a considerable amount of power on centimeter waves by the use of high frequency sparks, and in one case a radiation of fifty watts on a wave-length of 28.6 centimeters has been obtained,[11] using shock excitation from high frequency sparks.

REFLECTORS AND BEAM SYSTEMS.

An extensive series of investigations carried out at Tohoku Imperial University, Sendai, Japan, have demonstrated the practicability of using parabolic reflectors and wave directors on wave-lengths below five meters.[10,12] The above experiment on 41 centimeters by the same investigators shows that the types of reflectors and directors already developed can be very readily employed, and with great advantage, in conjunction with oscillators producing waves less than 100 centimeters in length.

POSSIBILITIES.

As regards the present applications and future possibilities of these high frequencies of the order of 500 megacycles, there is at present no practical use being made of them. One possibility is their use as a direction finding aid to navigation, either water or air, this being suggested by the fact that the

size of a complete directive transmitter and reflecting antenna system is comparable with the dimensions of a large size searchlight.

Considerable interest has been aroused in medical circles lately as to the possibilities of using intense fields of frequencies of this order, for the treatment of certain maladies. Some work in this direction is now in progress, and investigations are being carried out on the effect of these frequencies on various solutions, such as blood solutions.

A couple of years ago the General Electric Company built a 5 meter oscillator using about ten kilowatts, which produced some remarkable effects. Meters in the neighboring rooms were burnt out; a standing arc was produced at the end of a single rod in resonance with the transmitter; sausages could be cooked by simply hanging them over this rod; observers noted that their body temperature rose and found that the temperature of the blood had actually been made to rise slightly.

Bearing these facts in mind it will be of interest to observe what phenomena are produced when we obtain a few kilowatts of high frequency energy oscillating on a fifty centimeter wave-length.

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The Effect of Drawing on the Temperature Coefficient of the Electrical Resistivity of Constantan. R. S. J. SPILSBURY. (*Jour. Sci. Instr.*, Nov., 1929.) The author has found the temperature coefficient of electrical resistance of commercial constantan (copper-nickel) wire to vary from a positive value of 60 millionths per degree Centigrade to a negative value of 80 millionths. To get wire with a coefficient not greater than 10^{-6} a search had to be made for a sample meeting this requirement. Composition and heat treatment together do not fix the coefficient. A wire having a negative coefficient of 4^{-6} was drawn down from .03 in. to .004 in. and then annealed at 800° C. Its coefficient, measured at intermediate diameters gradually sunk to -26^{-6} per degree. Another sample having a positive coefficient of 58^{-6} when its diameter was .036 in., upon being drawn to .004 in. and annealed at 600° C., suffered a reduction of its coefficient to 31^{-6} per degree. In both cases the effect of drawing was to reduce the value of the coefficient. Whatever the cause may be it is not due to the production of a surface layer during annealing because cleaning the surface with emery paper causes only a slight change in the coefficient. "The principal practical deduction is that where fine wire is required it is necessary to start with material of positive coefficient. This is somewhat unfortunate, as the bulk of the commercial material shows a negative value." G. F. S.

The Rate of Transformation of Radium D. MRS. PIERRE CURIE AND MISS IRENE CURIE. (*J. de Phys. et le Rad.*, Nov., 1929.) The time required for radium to diminish to half of its original quantity is taken as 16.5 years. This was obtained from a study of the rate at which polonium formed in radium D initially free from it. For fifteen years from April, 1910, to January, 1926, observations have been in progress bearing on the period in question. From these results the conclusion that the previously adopted period is too small and that it is at least 19 years.

In a paper recording experiments made by herself but by a different method Miss Irene Curie obtains about 23 years as the half-period. The cause of the two different results, 19.5 years and 23 years is not known. G. F. S.

CORONAL DISCHARGE.

BY

H. A. GOULD,

Member of the Institute.

INTRODUCTION.

The tremendous industrial development of the country and the increasing use of power for domestic purposes create a continuous and rapidly growing demand for electric energy. When available power sites within close proximity of the distribution centers become developed to the economical limit, the increasing demand for power makes it necessary to go to distant points for the development of natural resources. It therefore requires the transmission of large amounts of power safely and economically over greater distances from generating stations, as well as the interconnection between existing systems for exchange of power from one system to another. Obviously higher voltages must be employed on the transmission lines.

The cost of transmission of electric energy increases with the length of the line. Long high tension transmission lines, outside of their actual cost, involve many other problems such as stability, regulation and losses. Among the losses in transmission of electric energy the loss due to coronal discharge is the subject of greatest uncertainty. The engineer can design a transmission line which he knows will be effective if it is constructed according to his specifications. He can make exact calculations only when he has a quantitative knowledge of all phenomena involved. It is therefore of great importance to have an exact knowledge of coronal losses in the transmission of power. In spite of the enormous quantity of experimental results and attempts at theoretical analyses of coronal discharge, there is a wide divergence of view and lack of a common ground among the leading research engineers and scientists. The object of this paper is to review the existing conceptions of the nature, mechanism and effects of coronal discharge on high tension transmission.

VISUAL CORONA.

When either a direct or alternating potential is applied between two smooth cylindrical conductors there is at the surface of the wire a high gradient of potential or electrical intensity. When this electrical intensity becomes great enough the air surrounding the conductor breaks down. If the applied potential is gradually increased a hissing noise is heard, and if the space is dark a pale violet light can be seen surrounding the conductor. A further increase of the potential enlarges the region of luminosity and if a watt meter is inserted in the circuit a loss of energy is noticed. This loss increases very rapidly as the voltage is raised. The glow, or breakdown, starts first near the conducting surface and expands with further increase of voltage until a spark strikes between the wires.

The term corona is usually taken to mean the glow or brush discharge which forms on conductors when the electric stress in their neighborhood is sufficient to cause local breakdown of the air. According to some writers, however, corona is considered to denote only the brush discharge, while others use the term to cover all forms of partial breakdown in air, namely the glow, brush and dark discharges.

Whenever corona is present there is always the characteristic odor of ozone. The oxygen of the air, when overstressed, becomes chemically very active and combines with metal, organic matter, etc. If the electrical stress of air surrounding the conductor is very high the oxygen enters into chemical combination with the nitrogen, forming oxides. The loss of energy by corona is thus in a number of forms, as sound, light, chemical action, heat, etc.

There is a distinct difference between the discharge around the positive and negative conductor. The positive discharge appears as an even bluish white glow extending to a definite distance from the conductor depending on the curvature, electric stress and pressure of the atmosphere. On negative conductors, the glow presents a beaded appearance in the form of more or less uniformly spaced reddish tufts. Alternating current corona examined through a stroboscope, so that one conductor is always seen when at the same half of the wave, has exactly the same appearance as the corresponding direct current corona.

IONIZATION OF AIR.

Since the early development of electrical science, it has been known that dry air is a non-conductor of electricity. A further study of electrical properties of matter has shown that conductivity is not a property of the matter itself but rather of its physical state. Mercury, for example, in a liquid form is a good conductor while cold mercury vapor is non-conducting. The leak or escape of electricity from an insulated charged body long ago attracted the attention of physicists. It was discovered that in the normal state the conductivity of air is very small; it may, however, in various ways be put into a state in which it easily becomes conductive. The application of such agents as X-rays, radioactive matter, matters undergoing chemical change such as the flame of a Bunsen burner, ultra-violet rays, etc., render the air electrically conductive. Air in a conducting state possesses certain characteristic properties. It retains its conductivity for some little time after the agent which made it a conductor has ceased to act.

The possibility of removing conductivity of the air by the application of an electric field indicates that something charged with electricity moves with the action of the field. Since the air, when in the conducting state, shows as a whole no charge of electricity the charges removed must be both positive and negative. We are thus led to the conclusion that the conductivity of the air is due to electrified particles mixed with the air, some of these particles having positive charges, others negative. We agreed to call these electrified particles "ions" and the process by which air is made conductive, "the ionization of air." (These ions are not identical with those met in the electrolysis of solutions.)

Experiments have shown that there is always a certain number of free ions in the atmosphere surrounding the earth. When low potential is applied between two conductors any free ions in the field are set in motion and a current may be detected by a highly sensitive instrument such as an electrometer. The usual laboratory galvanometer would not detect it since ionization current may be of a magnitude of 10^{-15} amperes. By gradual changing of the potential and reading of the magnitude of the ionization current a curve can be drawn such

that the ordinates represent the current through the air and the abscissæ the potential difference between the conductors. As long as the difference of the potential is small the curve is approximately a straight line, and in this stage the conduction obeys Ohm's law. The current however soon begins to increase more slowly than the potential and we reach a stage where there is no appreciable increase of current when the potential difference is increased; in this stage the current is said to be saturated.

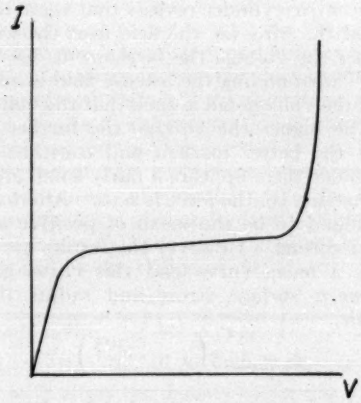
When parallel plates are used in the experiment the saturation current depends upon the amount of ionization between the plates. If the ionization takes place throughout the whole volume of air between the plates then the greater the distance between the plates the larger is the saturation current. Thus the behavior of the conducting air is very different from that of a metallic or liquid electrolytic conductor, for if such conductors were substituted for the air the greater the distance between the plates the smaller would be the current. When the potential difference is increased to such an extent that the electrical field is strong enough to ionize the air another stage is reached in which the current increases very rapidly with the potential difference, bringing about the well known effects of corona and spark.

The characteristic volt-ampere curve shown on Fig. 1 indicates three different stages of ionization. In the first stage the current increases with the electromotive force. The rate of increase diminishes as the force increases and the current tends to attain a maximum value. In the second stage the current remains practically constant and shows only small variations for large changes in electromotive force. The constant value is attained when the force is sufficiently great to collect all the positive and negative ions on the electrodes. In the third stage when the electromotive force is still further increased there is a sudden large increase in the ionization current when the electric field approaches the intensity required to produce corona.

Experiments with X-rays and Cathode rays passed through the air between the two conductors show that the air becomes a conductor, that is it becomes ionized. It appears that little particles are shot through the air at such high speed that they

break into pieces the molecules through which they pass, leaving ions to mark their trails. Hence we see that when very rapidly moving ions pass through the air they apparently come into collision with its molecules, ionizing the air; the energy required for the ionization is derived therefore from the kinetic energy of the rapidly moving ions. Inasmuch as the ionization of a molecule of air requires the expenditure of a finite amount of work, a moving ion cannot ionize a molecule

FIG. 1.



against which it strikes unless its kinetic energy exceeds a certain critical value. When this critical value is reached, a certain fraction of the number of collisions between the ions and the molecules will result in ionization. If, therefore, the ions are moving in an electric field the kinetic energy acquired by the ions will increase as the strength of the field increases. When the field is strong enough to make the kinetic energy of the ions exceed the critical value, the ions will acquire such velocities in their free paths that on colliding with neutral molecules many of the latter will split into pairs of ions, positive and negative. These new ions will themselves produce others by collision and so the current rapidly in-

creases, ultimately causing the characteristic corona effect. Thus the only feasible theory so far advanced to explain the mechanism of breakdown of the air is that of ionization by collision.

FIELD IN CORONA.

Studies of corona discharge were made with wire and concentric cylinder, parallel wires, sphere gaps and needle or point gaps. Most of the general theory has been worked out by taking the case of the wire and concentric cylinder, but it also can be applied to the other cases. Field explored for wire and concentric cylinder reveals that there is a sharp fall of potential at the wire, i.e. the field near the wire is intense and the higher the voltage the farther out does this intense field extend. Surrounding the intense field is a large volume within which the voltage fall is such that the voltage is nearly constant. The higher the voltage the further this portion extends, and the better marked and constant is the field. Near the cylinder there appears a fairly small area where the voltage is constant, i.e. the force is zero. Alternating current corona was found to be the mean of positive and negative direct current corona. However the results are very closely expressed by a mean curve and this curve gives for the relation between surface stress and radius the following formula for air,

$$g_s = 30.5\left(1 + \frac{0.305}{\sqrt{r}}\right),$$

where g_s is the stress at wire surface in kilovolts per centimeter and r is the radius of wire in cm.

The critical gradient for the formation of corona on parallel wires has been investigated by a great many engineers and physicists and the most accurate results may be expressed by the following formula

$$g_s = 30\left(1 + \frac{0.301}{\sqrt{r}}\right) \text{kv/cm.}$$

It was found that the surface of the wire has a great deal of influence upon the corona formation at high potentials.

Corona forms with difficulty on spheres. The gradient at the surface is a linear function of the reciprocal of the square

root of the radius for a given value of the ratio spacing to radius,

$$g_s = A + \frac{B}{\sqrt{r}},$$

where A and B are constant coefficients, the value of A being between 30 and 31 KV. per cm. for values of ratio spacing to radius approximately equal two.

The first discharge to take place from a point of a needle gap is a dark discharge. When the potential is sufficiently raised this turns into corona which however may not be stable. Thus if the point is too near the opposite electrode the discharge becomes intermittent and brushes form. It was found that the potentials are less the sharper the points, and are less for points with flat ends than for those with rounded ends, due to a greater flux concentration at the edges.

On the whole we see that the corona gradient depends only on the curvature of the surface at which corona forms. Accordingly the effect of pressure may be conveniently summarized as causing a change in density which affects the discharge in such way that the corona gradient is a function of density multiplied by the radius of curvature

$$g_s = 30p\left(1 + \frac{0.301}{\sqrt{pr}}\right),$$

where p is atmospheric pressure. Temperature has no effect except so far as it alters the density of the gas.

DIRECT CURRENT CORONA.

A scientific way to treat mathematically direct current corona discharge about a round wire is by determining the distribution of electric space-charge density and the voltage gradient as functions of the distance from the geometric axis of the conductor. Such treatment nevertheles requires the introduction of certain simplifications and assumptions. It involves the use of the divergence theorem and the specific velocities of ions.

The earliest systematic measurements of the velocity of ions in an electric field were made in 1897. By specific velocity of ions is understood the velocity in centimeters per

second with which ions will move through air under a gradient of one volt per centimeter. It was discovered that the velocities of positive and negative ions differ considerably. It was also found that specific velocity increases as the air pressure decreases. For dry air specific velocity at atmospheric pressure was found to be

$$K_+ = 1.36, \qquad K_- = 1.87.$$

Since the positive ions are considerably heavier and less mobile than the negative ones we may assume that in normal atmospheric pressure only negative ions are involved in the formation of the discharge and we can therefore write the following relations

$$i = K\rho g, \tag{1}$$

$$div g = 4\pi\rho c^2 10^{-9}, \tag{2}$$

where i is the current density in amperes per centimeter square, K is the specific velocity of ions in cm. per second under a gradient of one volt per cm., ρ is the space-charge density in coulombs per cu. cm., g is the electric field gradient in volts per cm., and c is the velocity of light $(c = 3 \times 10^{10}$ cm./sec.). Assuming a steady state and therefore no possibility of accumulation of charges we can write

$$div i = 0. \tag{3}$$

Now substituting equation (1) into equation (2) we have

$$4\pi\rho c^2 10^{-9} = div \frac{i}{K\rho};$$

but

$$div \frac{i}{K\rho} = \frac{1}{K\rho} div i + i \cdot \nabla \frac{1}{K\rho}.$$

Substituting equation (3) we get

$$div \frac{i}{K\rho} = 0 + i \cdot \nabla \frac{1}{K\rho}.$$

Hence

$$4\pi\rho c^2 10^{-9} = i \cdot \nabla \frac{1}{K\rho}. \tag{4}$$

Assuming a thin thread of an equal cross section throughout its length we can represent the scalar product

$$i \cdot \nabla \frac{1}{K\rho} = i \cdot \left(\frac{1}{K} \frac{d\frac{1}{\rho}}{dn} \right),$$

where dn is an element of the path in the direction of the current thread. This may also be written as

$$i \cdot \nabla \frac{1}{K\rho} = i \cdot \left(\frac{1}{K\rho^2} \frac{d\rho}{dn} \right). \tag{5}$$

Substituting equation (5) into equation (4) we get

$$4\pi K c^2 10^{-9} \frac{dn}{i} = \frac{d\rho}{\rho^3}.$$

Integrating this equation we get

$$4\pi K c^2 10^{-9} \int \frac{dn}{i} = \frac{1}{2\rho^2} + C. \tag{6}$$

Here the integral $\int \frac{dn}{i}$ must be taken along the thread and C is an integration constant which can be determined from the initial conditions.

Solving equation (6) we find the value of

$$\rho = \frac{1}{\sqrt{8\pi K c^2 10^{-9} \int \frac{dn}{i} + 2C}} \tag{7}$$

and from equation (1)

$$g = \frac{i}{K} \sqrt{8\pi K c^2 10^{-9} \int \frac{dn}{i} + 2C}. \tag{8}$$

For the surface of the wire

$$n = r, \qquad g = g_s, \qquad i = i_s;$$

hence $\int \frac{dn}{i} = 0$ and therefore

$$g_s = \frac{i_s}{K} \sqrt{2C}.$$

Now substituting this value in equation (7) we get the equation for the space-charge density

$$\rho = \frac{1}{\sqrt{8\pi K c^2 10^{-9} \int_r^x \frac{dn}{i} + \left(\frac{g_s}{i_s} K\right)^2}} \tag{9}$$

and for field gradient

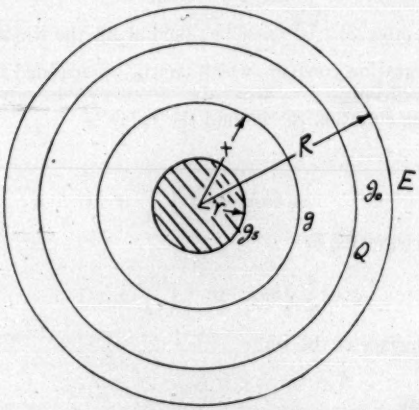
$$g = i \sqrt{\frac{8\pi c^2 10^{-9}}{K} \int_r^x \frac{dn}{i} + \left(\frac{g_s}{i_s}\right)^2}. \tag{10}$$

These relations are for any point x on the thread expressed in terms of known quantities.

ALTERNATING CURRENT CORONA.

The theory of A.C. corona is much more involved than that of D.C. corona since the space charge and the velocities of ions are then functions not only of the distance from the axis of the

FIG. 2.



conductor but of time as well. Moreover, the phenomenon becomes dependent on the frequency of the supply, since an ion which at low frequencies may travel during one alternation

from the inner wire to the outer cylinder, at high frequency may be caught and reversed near the inner conductor. A somewhat simplified analysis of the conditions about a cylindrical conductor may be described by the use of the cross section of such a conductor shown in Fig. 2. The shaded portion represents the conductor whose radius is r. The gradient at the surface of this conductor is g_s, and g represents the electrical gradient at any point distant x from the center of the conductor. The cylindrical surface Q, whose radius is b, represents the surface where the electrical gradient has the constant value g_0. This gradient g_0 is the least gradient of potential at which ionization by impact can occur. Ionization by impact and the coronal discharge can take place only between the surface Q and the wire, in which region the electrical intensity is greater than g_0. Let E represent the potential difference between the wire and the ground or some distant conductor, and let the distance of this other conductor or ground be R.

The electrical gradient at x is:

$$g = \frac{E}{x \log\left(\frac{R}{r}\right)}.$$

At the surface of the wire it is:

$$g_s = \frac{E}{r \log\left(\frac{R}{r}\right)}.$$

At surface Q it is:

$$g_0 = \frac{E}{6 \log\left(\frac{R}{r}\right)}.$$

From the above we obtain:

$$g = g_0 \frac{6}{x}$$

and

$$g_s = g_0 \frac{6}{r}.$$

Assuming that at atmospheric pressure only negative ions can ionize by impact the physical process involved in the establishment of corona may be traced in the following way: The conductor is subject to an alternating potential. Let the maximum value of this at crest of the wave be just sufficient to produce the corona. Whether it is due to the radioactive radiations from the earth or some other cause, there is an initial ionization of air and therefore a certain number of ions in the region surrounding the wire. Suppose the wire to be at the positive phase, the initially ionized particles move inward and when they cross the surface Q they begin to ionize by collision producing a crop of new ions all of them approaching the wire. Here some of these ions disappear, resulting in a current loss. On the next phase the wire is negative, and the ions move outward from the wire. Some of them disappear by recombination but there is left a much greater number than were there originally. These ions move again inward on the positive phase and thus the process goes on until after a number of cycles the density of ionization about the wire at crest of positive phase is n. Now let n_0 be the number of ions in a unit time crossing the surface where the intensity is g_0 and assume that the corona appears when n reaches a fixed constant value. Let α be the number of ions produced by one negative ion in going one centimeter.

If the ion moves a distance dx the number produced will be αdx and if there are N such ions crossing any cylindrical surface, the number crossing a surface dx distant will be

$$dN = \alpha N dx.$$

On integrating this between n_0 and n we have

$$\frac{n}{n_0} = e^{\int_r^b \alpha dx}.$$

The number of ionizing impacts per centimeter indicated by α depends upon the value of the electrical gradient g, which in turn depends on the distance x from center of conductor. Consequently α will depend upon x. Also α depends upon the pressure of the air. Since we find that α is a function of x and p, it is necessary to express the form of this functional

relation. However the mathematical relation obtained for these functions is quite involved and not directly integrable for x. An integral equation which is a close approximation to the above was proposed and the curve plotted is in good agreement with the empirical equation expressing the experimental relation obtained for the gradient at the surface of the wire, radius of the wire and pressure or density of the air on the appearance of the corona. This relation is in no way connected with the important problem of loss of energy from the conductor, this loss increasing very rapidly as the potential is increased beyond that required to produce corona.

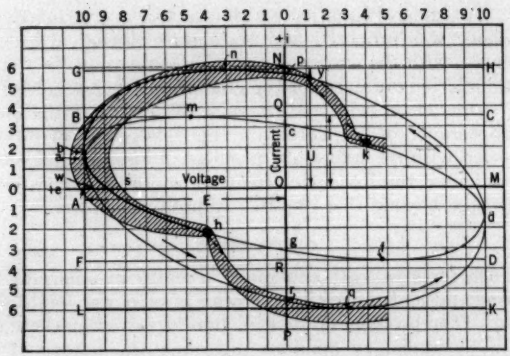
CORONA LOSS.

Much progress has been made recently in enlarging the experimental and theoretical knowledge of power loss due to corona. The problem however is hardly solved as there still remain contradictory opinions regarding the nature of the mechanism of the phenomenon. The theoretical development of the laws of corona formation is based on the mobilities of ionized air particles, yet there is no assurance that all mobilities found in experimentations are not to be attributed to metallic particles, atoms or clusters of metallic molecules that are charged and formed by the disintegration of the wire by corona discharge. The field in corona still presents an object for further investigation and studies.

Power loss due to corona is still a matter of serious investigations, experimental and theoretical. It has been found that weather conditions have a serious effect on power loss. Moisture increases the loss considerably while snowstorms increase the loss several times. Rain has a very strong effect, glows turning into brushes when the wires become wet. Wind has no effect but smoke produces a slight increase of loss. High frequencies appear to decrease the power loss at high voltages. No law of power loss has yet been advanced which would prove satisfactory for all conditions of coronal discharge.

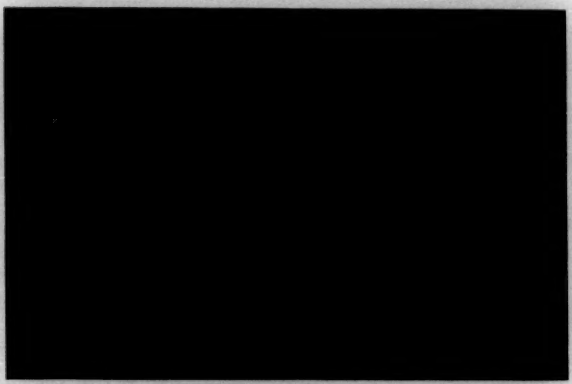
The application of new methods such as the cathode ray oscillograph has made it possible to study the alternating current corona in much greater detail quantitatively from instant to instant, thus bringing us nearer a rational explana-

FIG. 3.



An actual oscillogram of A.C. Corona (Peek) with the corresponding ellipses.

FIG. 4.



Corona on copper wire polished after each exposure (Peek). Diameter 0.186 m.

tion of the ionic mechanism involved. Cyclograms of a.c. corona made by means of a cathode-ray oscillograph have recently been obtained, in which horizontal distances from the vertical axis are proportional to the instantaneous values of the sinusoidal applied voltage, the vertical ordinates representing the corresponding instantaneous values of the charging and loss current flowing into the wire (Fig. 3). The cathode beam traces a diffused figure representing an ellipse with the amplitude of the voltage and the current as the principal semi-axis as long as the sinusoidal amplitude of the applied voltage is below the visual corona point. When, however, the minimum ionization voltage is exceeded during a part of each alternation the cyclogram ceases to be an ellipse but consists of four portions per cycle, two of which correspond to the intervals of time during which the corona is extinct, and the other two when corona is present, with quite short transients in between.

In order to explain the mechanism of corona formation and the influence of the space-charge upon the instantaneous critical voltage, an artificial corona was produced by using two condensers in series, one of which was imperfect, representing the ionized portion of the air adjacent to the wire, the other a perfect condenser representing the remote portions of the air in which no ionization by collisions takes place.

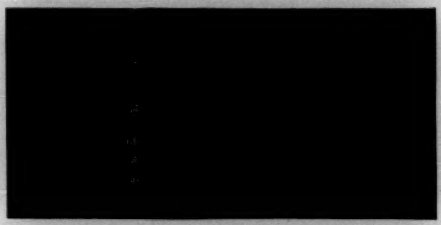
Strictly speaking, the actual conditions in an alternating current corona can not be adequately represented by an equivalent diagram of condensers, fixed glow gaps, resistances, etc., such as were used in these experiments, since there is in reality no definite stationary cylindrical surface, concentric with the wire, which could serve as a boundary between some fictitious condensers such as were used in the experiment. However the obtained diagram leads to a cyclogram which checks well with the experiment. A mathematical theory of the observed cyclograms was developed by Karapetoff and approximate equations were derived for current and voltage as functions of time.

In reviewing the subject no consideration has been given to the nature and mechanism of luminosity due to corona. It is evident that particles become unstable due to the change of their internal energy. An analysis of this phase of the

FIG. 5.

(1) without stroboscope.

Left (+). (2) with stroboscope. Right (−).



Left (−). (3) with stroboscope rotated at 180°. Right (+).

Corona on parallel wires. No. 13 B. & S. copper wire. Spacing 12.7 cm. Volts 82,000 (Peek).

phenomenon would be a subject outside of the scope of this work. The fact remains that the progress of our knowledge of the mechanism of corona is closely related to that of the recent research in the general field of nature of matter. There is no doubt about this knowledge advancing, not continuously but by quantum jumps, so to speak. In general the procedure of scientific research is first to experiment roughly then to find improved and more refined methods of studying. The next step is generalization in the form of an hypothesis and theory which enables in turn to make predictions. This is the time when difficulties arise puzzling the research workers for years, urging to find ways of surmounting them until suddenly new light is thrown because of some new experimental or theoretical development that may be in an adjacent apparently disconnected field. The two fields are pulled together into a new and broader picture to start the advanced march over again, attacking the problem from an entirely different angle and view point. Such may possibly be the case in the study of the mechanism of coronal discharge and losses.

Elements Unidentified or Doubtful in the Sun. CHARLES E. ST. JOHN. (*Astrophys. J.*, Oct., 1929.) To prove that an element exists in the sun is it enough to show that an absorption line in the sun's spectrum has the same wave-length as a line of the element produced on the earth? "We know now that exact coincidence is the exception and, when exact, points to possible errors of observation. As a rule, the departures from coincidence are positive for λ sun *minus* λ arc and of the order of 0.01A. Though close agreement in wave-length offers usually a first line of attack, this is not always the case. Not infrequently the approach has to come through an analysis of the spectrum which has shown just what to look for and where to look; for example the oxygen doublet in the infra-red. Identifications of elements represented by few solar lines resting upon coincidences alone are subject to much uncertainty through accidental agreements in wave-length. In such cases considerations based upon atomic theory have been the determining factors."

Of the 90 known elements 58 are listed as existing in the sun, though the strength of the evidence is by no means the same for all. The following are not known to exist in the sun, argon, actinium, arsenic, bismuth, bromine, chlorine, fluorine, gold, iodine, iridium, krypton, masurium, mercury, neon, osmium, phosphorus, protactinium, polonium, radium, rhenium, radon, selenium, tantulum, tellurium, thorium, uranium, xenon and the rare earths, holmium, illinium, lutecium, terbium and thulium.

Radon, radium emanation, has an average life of 5.57 days. There is no reason to look for it in the sun before its progenitors, thorium, uranium and radium, have been found there. The halogens, fluorine, chlorine, bromine and iodine, all are missing from the sun and stars. In view of their abundance on the earth this would be surprising, were there not reason to believe that the ozone in the upper atmosphere absorbs the light that might prove their presence. Nitrogen seems to exist in large quantities in the sun. G. F. S.

DYNAMICAL ANALYSIS OF MACHINES.*

BY

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DYNAMICS OF MECHANISMS.

We will now consider the dynamical analysis of the mechanisms already considered in the kinematic procedures; viz., the four bar linkage, the sliding block linkage and the swinging block and rotary engine mechanism. In general we have a driving force or torque, either applied internally as the steam pressure in the reciprocating mechanism, or externally as in the drive shaft torque of the four bar linkage. This is resisted by the resistance of the driven shaft; that is, the reacting torque on the driven shaft. The driving force or torque is usually some function of the angular coördinate, the resisting torque is either constant (approx.), or a function of the coördinate. In the case of an elastic connection at the driven shaft, the resisting torque is proportional to the angular displacement.

Elementary Engine Mechanism (Fig. 32).—Neglecting the obliquity of the connecting rod, we have, approximately:

$$s = r(1 - \cos\theta); \qquad \therefore \quad ds = rd\theta \sin\theta;$$

$$\therefore \quad \frac{ds}{d\theta} = r \sin\theta = \frac{v}{\dot{\theta}}, \quad \text{where} \quad \left| \begin{array}{l} \dot{s} = v \\ \dot{\theta} = w. \end{array} \right.$$

To find the torque on the shaft equivalent to the reciprocating kinetic reaction:

* Extension of a portion of a dissertation for the degree of doctor of philosophy submitted to Clark University, 1928.

Continued from p. 405, vol. 209, March, 1930.

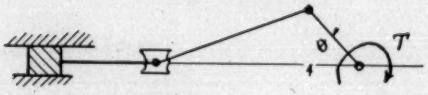


FIG. 32

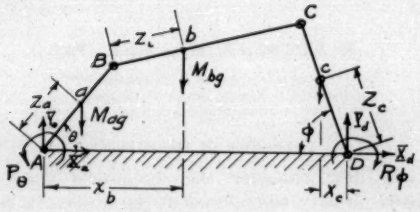


FIG. 33

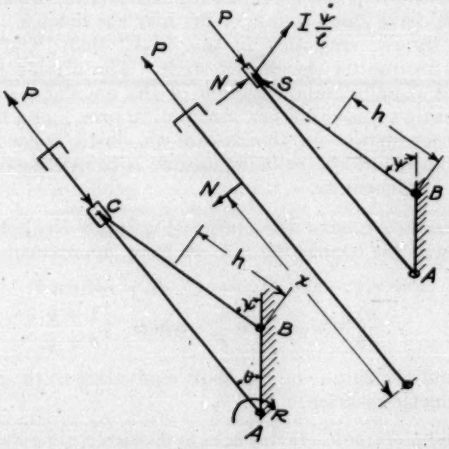


FIG. 34

The quivalent inertia coefficient of the reciprocating parts referred to the shaft, is:

$$A = m\left(\frac{ds}{d\theta}\right)^2 = mr^2 \sin^2 \theta$$

and

$$\frac{dA}{d\theta} = 2mr^2 \sin \theta \cos \theta.$$

Hence, the dynamic equation for the torque is:

$$mr^2 \sin^2 \theta \cdot \ddot{\theta} + mr^2 \sin \theta \cos \theta \omega^2 = T.$$

By the method of velocity ratios:

$$K = \frac{v}{\omega} = r \sin \theta, \qquad \frac{dK}{d\theta} = r \cos \theta;$$

$$\therefore \quad mK^2\ddot{\theta} + mK\frac{dK}{d\theta}\omega^2 = T,$$

$$mr^2 \sin^2 \theta \cdot \ddot{\theta} + mr^2 \sin \theta \cos \theta \omega^2 = T.$$

Four Bar Linkage Mechanism (Fig. 33).—We will assume the driving shaft driven by an external torque P_θ and the driven shaft resisted by the torque reaction on the driven shaft R_ϕ.

The kinetic energy of the system is:

$$T = \tfrac{1}{2}(I_a\dot{\theta}^2 + I_b\dot{\psi}^2 + I_c\dot{\phi}^2 + M_b(\dot{x}_b^2 + \dot{y}_b^2),$$

where I_a = moment of inertia of crank "a" about A,
I_c = moment of inertia of the oscillating lever "c" about D,
I_b = moment of inertia of connecting rod "b" about its c. of g.,
x_b and y_b are the coördinates of the c. of g. from A,
z_L = distance of c. of g. from hinge B,
z_a = distance of c. of g. of crank system from axis A,
z_c = distance of c. of g. of oscillating lever from axis D.

We may express T in terms of θ and the velocity ratios:

$$\tfrac{1}{2}A\dot{\theta}^2 = \tfrac{1}{2}[I_a + I_bK_{\psi\theta}^2 + I_cK_{\phi\theta}^2 + M_b(K_{bx\theta}^2 + K_{by\theta}^2)]\dot{\theta}^2,$$

so that the coefficient of inertia of the system is:

$$A = I_a + I_bK_{\psi\theta}^2 + I_cK_{\phi\theta}^2 + M_b(K_{bx\theta}^2 + K_{by\theta}^2).$$

The generalized equation of motion of the mechanism is:

$$A\ddot{\theta} + I_b K_{\psi\theta}\frac{dK_{\psi\theta}}{d\theta} + I_c K_{\phi\theta}\frac{dK_{\phi\theta}}{d\theta} + M_b\left(K_{x\theta}\frac{dK_{x\theta}}{d\theta} + K_{y\theta}\frac{dK_{y\theta}}{d\theta}\right)$$
$$= P_\theta - R_\phi K_{\phi\theta} - M_a g Z_a \cos\theta - M_b g K_{y\theta} - M_c g z_c \cos\phi \cdot K_{\phi\theta},$$

where the velocity ratios and their derivatives have been derived from the kinematic procedure. It is not profitable at this time to make the substitutions, since we are more concerned now with outling a procedure of analysis with specific illustrations, rather than the numerical solution of any given problem.

External Reactions—Reactions on the Frame (Fig. 33).—The external reactions consists of the bearing reactions at A and D and the torque reactions on the shafts P_θ and R_ϕ. The bearing reactions are the components of the frame reaction on the mechanism, which resolves into the torque reaction R_f and the translation components $\sum X_f$ and $\sum Y_f$ of the frame. We note the relations:

$$\sum X_f = X_A + X_D, \qquad Y_f = Y_A + Y_D$$

and

$$R_f = x_d Y_D - y_d X_D,$$

where x_d and y_d are the coördinates of the bearing reaction D with respect to the axis A.

The frame reactions on the mechanism are:

$$\sum X_f = (m_a K_{ax\theta} + m_b K_{bx\theta} + m_c K_{cx\theta})\ddot{\theta}$$
$$+\left(m_a\frac{dK_{ax\theta}}{d\theta} + m_b\frac{dK_{bx\theta}}{d\theta} + m_c\frac{dK_{cx\theta}}{d\theta}\right)\dot{\theta}^2,$$

$$\sum Y_f = (m_a K_{ay\theta} + m_b K_{by\theta} + m_c K_{cy\theta})\ddot{\theta}$$
$$+\left(m_a\frac{dK_{ay\theta}}{d\theta} + m_b\frac{dK_{by\theta}}{d\theta} + m_c\frac{dK_{cy\theta}}{d\theta}\right)\dot{\theta}^2,$$

$$R_f = [m_a(x_a K_{ay\theta} - y_a K_{ax\theta}) + m_b(x_b K_{by\theta} - y_b K_{bx\theta})$$
$$+ m_c(x_c K_{cy\theta} - y_c K_{cx\theta}) + I_a + I_b K_{\psi\theta} + I_c K_{\phi\theta}]\ddot{\theta}$$
$$+\left[m_a\left(x_a\frac{dK_{ay\theta}}{d\theta} - y_a\frac{dK_{ax\theta}}{d\theta}\right) + m_b\left(x_b\frac{dK_{by\theta}}{d\theta} - y_b\frac{dK_{bx\theta}}{d\theta}\right)\right.$$
$$\left.+ m_c\left(x_c\frac{dK_{cy\theta}}{d\theta} - y_c\frac{dK_{cx\theta}}{d\theta}\right) + I_b\frac{dK_{\psi\theta}}{d\theta} + I_c\frac{dK_{\phi\theta}}{d\theta}\right]\dot{\theta}^2 + R_\phi - P_\theta.$$

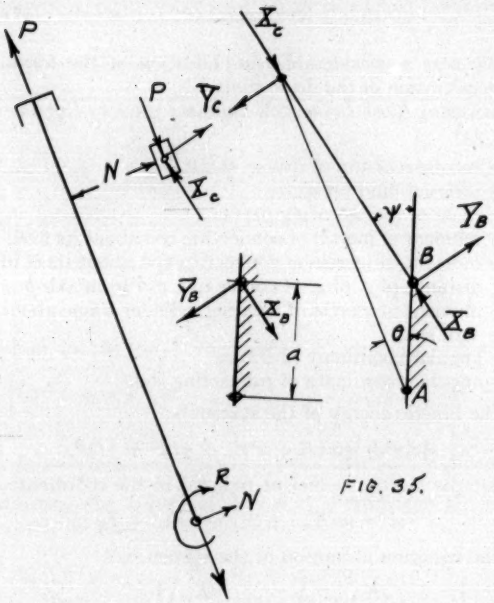


FIG. 35.

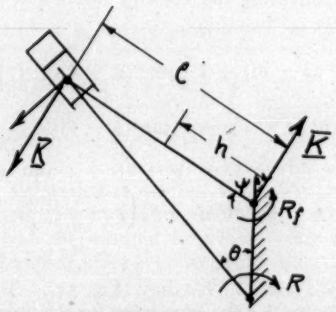


FIG. 36

We note a considerable simplification of the formulæ ... the acceleration of the drive shaft $\ddot{\theta}$ is nil.

Swinging Block Linkage Mechanism—Rotary Engine.—Let (Fig. 34)

R = torque reaction of driven shaft,
P = internal fluid pressure,
m = mass of reciprocating parts,
I = moment of inertia of connecting rod about its fixed axis,
I_g = moment of inertia of connecting rod about its c. of g.,
h = distance of c. of g. of connecting rod from axis b,
I_f = moment of inertia of rotating cylinder frame about axis of shaft,
θ = angular coördinate of frame,
ψ = angular coördinate of connecting rod.

The kinetic energy of the system is:

$$\tfrac{1}{2}A\dot{\theta}^2 = \tfrac{1}{2}m(\dot{x}^2 + x^2\dot{\theta}^2) + \tfrac{1}{2}I\dot{\psi}^2 + \tfrac{1}{2}I_f\dot{\theta}^2,$$

so that the inertia coefficient reduced to the coördinate θ is:

$$A = m(K_{x\theta}^2 + x^2) + IK_{\psi\theta}^2 + I_f.$$

The equation of motion of the system, is:

$$A\ddot{\theta} + \left[mK_{x\theta}\left(\frac{dK_{x\theta}}{d\theta} + x\right) + IK_{\psi\theta}\frac{dK_{\psi\theta}}{d\theta}\right]\dot{\theta}^2 = -PK_{x\theta} - R,$$

so that, on substituting the velocity ratios and their derivatives, we have:

$$\left[mx^2(1 + \tan^2(\psi - \theta)) + I\frac{x^2}{l^2}\sec^2(\psi - \theta) + I_f\right]\ddot{\theta}$$
$$-\left[m(x^2\tan^3(\psi - \theta)) - \frac{x^2}{l}\tan(\psi - \theta)\right.$$
$$\times \sec^3(\psi - \theta)(x - l\cos(\psi - \theta)) + x^2\tan(\psi - \theta)$$
$$\left. + I\frac{x^2}{l^2}\sec^2(\psi - \theta)\tan(\psi - \theta)\left(2 - \frac{x}{l}\sec(\psi - \theta)\right)\right]\dot{\theta}^2$$
$$= Px\tan(\psi - \theta) - R.$$

Analysis by Synthetic Procedure (Fig. 34).—To verify this equation let us consider the expression for the torque obtained from a direct analysis of the mechanism.

For the motion of the sliding block:

$$S\cos(\psi-\theta)+I\frac{\ddot{\psi}}{l}\sin(\psi-\theta)-P=m(\ddot{x}-x\dot{\theta}^2),$$

$$S\sin(\psi-\theta)-I\frac{\ddot{\psi}}{l}\cos(\psi-\theta)-N=m(x\ddot{\theta}+2\dot{x}\dot{\theta}),$$

so that the lateral thrust on the rotating frame is:

$$N=P\tan(\psi-\theta)+m[(\ddot{x}-x\dot{\theta}^2)\tan(\psi-\theta)-x\ddot{\theta}-2\dot{x}\dot{\theta}]-I\frac{\ddot{\psi}}{l}\sec(\psi-\theta).$$

Therefore, for the torque on the shaft R: since $Nx-R=I_f\ddot{\theta}$,

$$R=\left[(P+m(\ddot{x}-x\dot{\theta}^2))\tan(\psi-\theta)-m(x\ddot{\theta}+2\dot{x}\dot{\theta})-I\frac{\ddot{\psi}}{l}\sec(\psi-\theta)\right]x-I_f\ddot{\theta}.$$

Substituting the values of $\ddot{x}$, $\dot{x}$ and $\ddot{\psi}$ from the kinematic procedure, we have:

$$R=Px\tan(\psi-\theta)-\left[mx^2(1+\tan^2(\psi-\theta))+I\frac{x^2}{l^2}\sec^2(\psi-\theta)+I_f\right]\ddot{\theta}+\left[m\left\{x^2\tan^3(\psi-\theta)-\frac{x^2}{l}\tan(\psi-\theta)\sec^3(\psi-\theta)(x-l\cos(\psi-\theta))+x^2\tan(\psi-\theta)\right\}+I\frac{x^2}{l^2}\tan(\psi-\theta)\times\sec^2(\psi-\theta)\left(2-\frac{x}{l}\sec(\psi-\theta)\right)\right]\dot{\theta}^2.$$

Analysis by Method of Virtual Work.—A third procedure of general analysis is by the method of virtual displacements. In this method the internal reactions, other than the fluid pressure, disappear, since collectively they are non-work forces.

We will consider a single cylinder, connecting rod, etc., as typical of the entire mechanism of a rotary engine (Fig. 34).

The internal work forces are P and $-P$, which do the work $-P\dot{x}$ per unit time. The work of the kinetic reactions are $-m(\ddot{x}-x\dot{\theta}^2)\dot{x}$ and $-m(2x\dot{x}\dot{\theta}^2+x^2\dot{\theta}\ddot{\theta})$ for the reciprocating parts and $-Ig\dot{\psi}\ddot{\psi}$ and $-mh^2\dot{\psi}\ddot{\psi}$ for the connecting rod. The work of the kinetic reaction of the frame is $-I_f\ddot{\theta}\dot{\theta}$. The external torque reaction on the shaft does the work $-R\dot{\theta}$ per unit time. We have, therefore,

$$-P\dot{x}-m(\ddot{x}-x\dot{\theta}^2)\dot{x}-m(2x\dot{x}\dot{\theta}^2+x^2\dot{\theta}\ddot{\theta}) \\ -I\dot{\psi}\ddot{\psi}-I_f\ddot{\theta}\dot{\theta}-R\dot{\theta}=0;$$

hence,

$$R=-P\frac{\dot{x}}{\dot{\theta}}-m(\ddot{x}-x\dot{\theta}^2)\frac{\dot{x}}{\dot{\theta}}-2mx\dot{x}\dot{\theta}-mx^2\ddot{\theta}-I\dot{\psi}\frac{\dot{\psi}}{\dot{\theta}}-I_f\ddot{\theta}$$

$$=-P\frac{\dot{x}}{\dot{\theta}}-m\ddot{x}\frac{\dot{x}}{\dot{\theta}}-mx\dot{x}\dot{\theta}-I\dot{\psi}\frac{\dot{\psi}}{\dot{\theta}}-(I_f+mx^2)\ddot{\theta},$$

where $I=I_g+mh^2$. Since

$$\frac{\dot{x}}{\dot{\theta}}=K_{x\theta},\qquad \frac{\dot{\psi}}{\dot{\theta}}=K_{\psi\theta}\quad\text{and}\quad K_{\lambda\theta}=\frac{d\lambda}{d\theta}=x,$$

for $d\lambda=xd\theta$. Therefore, in terms of the reduced kinetic reactions and forces:

$$R=-PK_{x\theta}-m(\ddot{x}-x\dot{\theta}^2)K_{x\theta}-2m\dot{x}\dot{\theta}\cdot\dot{x} \\ -I\dot{\psi}K_{\psi\theta}-(I_f+mx^2)\ddot{\theta}.$$

From the kinematic relations, we note the velocity criterion gives:

$$-\frac{\dot{x}}{x\dot{\theta}}=\tan(\psi-\theta)\quad\text{and}\quad\frac{\dot{\psi}}{\dot{\theta}}=\frac{x}{l\cos(\psi-\theta)},$$

so that we may also write:

$$R=x(P+m(\ddot{x}-x\dot{\theta}^2)\tan(\psi-\theta))-2mx\dot{x}\dot{\theta} \\ -\frac{x}{l}\frac{I\dot{\psi}}{\cos(\psi-\theta)}-(I_f+mx^2)\ddot{\theta},$$

which on substituting for the accelerations and velocities in terms of the coördinate θ, gives the previous expressions.

REACTIONS ON THE FRAME.

We may proceed by directly calculating the reactions of the connecting rod at its fixed end and then take moments about the axis of rotation. This gives the reactive torque. A second method is based on the relation that the external torque on the shaft plus the reaction torque of the frame on the mechanism is equal to the rate of change of the angular momentum of the total mechanism about the axis of rotation. The torque reaction on the shaft is already determined from the generalized equation of motion, and in a general computation is usually estimated first. So the problem by this latter method requires the estimation of the moments of the kinetic reaction about the shaft. By the principle of superposition of forces, we may estimate the component torque effects on the frame for each part and then sum up these reactions for the total effect.

By the Direct or Synthetic Method (Fig. 35).—Considering first the reactions on the piston, we have (Fig. 35):

$$P + m(\ddot{x} - x\dot{\theta}^2) - Y_c \sin(\psi - \theta) - X_c \cos(\psi - \theta) = 0,$$

where $Y_c = \frac{I\ddot{\psi}}{l}$, so that,

$$X_c = \frac{Pm(\ddot{x} - x\dot{\theta}^2)}{\cos(\psi - \theta)} - \frac{I\ddot{\psi}}{l}\tan(\psi - \theta),$$

Therefore,

$$N = X_c \sin(\psi - \theta) - Y_c \cos(\psi - \theta) - 2m\dot{x}\dot{\theta} - mx\ddot{\theta}$$

and

$$R = \left[(P + m(\ddot{x} - x\dot{\theta}^2))x \tan(\psi - \theta) - \frac{I\ddot{\psi}}{l}\tan(\psi - \theta)\sin(\psi - \theta) - \frac{I\ddot{\psi}}{l}\cos(\psi - \theta)\right] x - 2mx\dot{x}\dot{\theta} - (I_f + mx^2)\ddot{\theta};$$

$$\therefore \quad R = (P + m(\ddot{x} - x\dot{\theta}^2))x \tan(\psi - \theta) - \frac{I\ddot{\psi}x}{\cos(\psi - \theta)} - 2mx\dot{x}\dot{\theta} - (I_f + mx^2)\ddot{\theta}.$$

For the reacting torque on the frame, we have (Fig. 35):

$$\begin{aligned} R_f &= (X_B \sin\psi - Y_B \cos\psi)a \\ &= (X_c - m_c\psi^2 h)a \sin\psi - (Y_c - m_c h\ddot{\psi})a\cos\psi \\ &= \frac{P + m(\ddot{x} - x\dot{\theta}^2)}{\cos(\psi - \theta)} a \sin\psi - \frac{I\ddot{\psi}}{l} a \tan(\psi - \theta)\sin\psi \\ &\qquad - m_c\psi^2 ha \sin\psi - \left(\frac{I}{l} - m_c h\right)\ddot{\psi}a\cos\psi; \end{aligned}$$

$$\therefore\quad R_f = \left(\frac{P + m(\ddot{x} - x\dot{\theta}^2)}{\cos(\psi - \theta)}\right) a \sin\psi - \left(\frac{\cos\theta}{\cos(\psi - \theta)}\frac{I}{l} - m_c h\cos\psi\right)\ddot{\psi}a - m_c\psi^2 ha\sin\psi.$$

By the Overall-Torque Method (Fig. 36).—In this case, we have (Fig. 36), $R_f - R$ = moment of kinetic reactions about the axis of rotation A − since the external torque $(-R)$ on the shaft is equal and opposite to the torque exerted by the mechanism on the shaft; that is, to the driving torque R. We note the fluid pressure on piston and its reaction on cylinder head are internal forces and have no resultant moment effect.

In calculating the moments of the kinetic reaction, it is always important to make such resolutions so that the kinetic moments may be estimated rapidly. In this connection, the advantage of the property of a couple should be noted, since its moment is independent of any axis.

The kinetic reaction of the connecting rod will be resolved into (1) a couple,

$$I_g\ddot{\psi} - m_c h(l - h)\ddot{\psi} = (I - m_c hl)\ddot{\psi},$$

where $I = I_g + m_c h^2$ and (2) an additional mass at the piston,

$$m' = m_c\frac{h}{l}.$$

Therefore, the moment of the kinetic reactions are:

$$2\left(m + m_c\frac{h}{l}\right)x\dot{x}\dot{\theta} + (I - m_c hl)\ddot{\psi} + \left(I_f + \left(m + m_c\frac{h}{l}\right)x^2\right)\ddot{\theta}.$$

Substituting the previously derived value of the torque, we obtain:

$$R_f = (P + m(\ddot{x} - x\dot{\theta}^2))x \tan(\psi - \theta) - \frac{I\ddot{\psi}x}{l\cos(\psi - \theta)} + 2m_c\frac{h}{l}x\dot{x}\dot{\theta} + (I - m_chl)\ddot{\psi} + m_c\frac{h}{l}x^2\ddot{\theta}.$$

Now by geometry, we can show:

$$\frac{a\sin\psi}{\cos(\psi - \theta)} = x\tan(\psi - \theta); \qquad x = a\frac{\sin\psi}{\sin(\psi - \theta)};$$

$$l = \frac{a\sin\theta}{\sin(\psi - \theta)} \quad \text{and} \quad x\cos(\psi - \theta) = l + a\cos\psi$$

and from the kinematic relation at the piston:

$$x\ddot{\theta} + 2\dot{x}\dot{\theta} = l\ddot{\psi}\cos(\psi - \theta) - l\dot{\psi}^2\sin(\psi - \theta),$$

so that the terms:

$$(P + m(\ddot{x} - x\dot{\theta}^2))x\tan(\psi - \theta) = \left(\frac{P + m(\ddot{x} - x\dot{\theta}^2)}{\cos(\psi - \theta)}\right)a\sin\psi,$$

$$I\ddot{\psi}\left(\frac{x}{l\cos(\psi - \theta)} - 1\right) = I\ddot{\psi}\frac{a\cos\theta}{l\cos(\psi - \theta)},$$

$$2m_c\frac{h}{l}x\dot{x}\dot{\theta} - m_chl\ddot{\psi} = m_cha\ddot{\psi}\cos\psi - m_cha\dot{\psi}^2\sin\psi - m_c\frac{h}{l}x^2\ddot{\theta}.$$

Therefore, the reacting torque on the frame is:

$$\begin{aligned} R_f &= (P - m(\ddot{x} - x\dot{\theta}^2))x\tan(\psi - \theta) \\ &\quad + \left(I - m_chl - \frac{Ix}{\cos(\psi - \theta)}\right)\ddot{\psi} + 2m_c\frac{h}{l}x\dot{x}\dot{\theta} + m_c\frac{h}{l}x^2\ddot{\theta} \\ &= \left(\frac{P + m(\ddot{x} - x\dot{\theta}^2)}{\cos(\psi - \theta)}\right)a\sin\psi \\ &\quad - \left(\frac{\cos\theta}{\cos(\psi - \theta)}\frac{I}{l} - m_ch\cos\psi\right)a\ddot{\psi} - m_cha\dot{\psi}^2\sin\psi, \end{aligned}$$

which verifies our previous derivation.

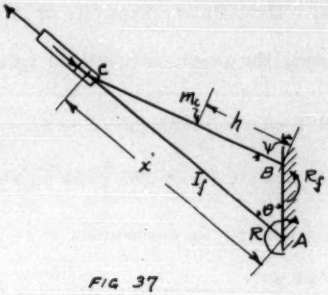


FIG. 37

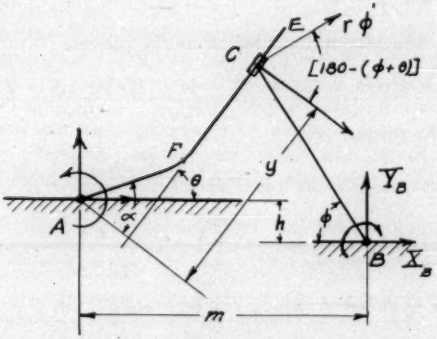


FIG. 38

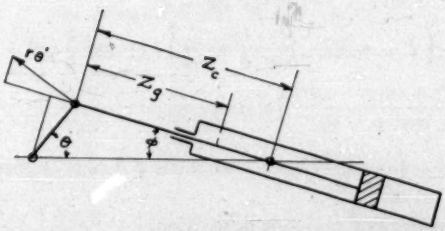


FIG. 39

Angular and Linear Momentum of System (Fig. 37).—The moment of the external reactions about the axis of A is equal to the rate of change of the angular momentum of the system about A. Considerable simplification is effected by considering the connecting rod reduced to two weighted masses at the ends together with a rotary inertia.

$$(I_g - m_c h(l - h)) = I - m_c hl,$$

$-R_f$ = Reaction on frame about propeller axis.

The angular momentum of the system about A is (Fig. 37):

$$\left(m + m_c\frac{h}{l}\right)x^2\dot{\theta} + (I - m_c hl)\dot{\psi} + I_f\dot{\theta},$$

where I is the moment of inertia of the connecting rod and I_f is that of the rotating frame. Let $K_{\psi\theta}$ be the angular velocity ratio of the connecting rod, $K_{x\theta}$ ond $K_{\lambda\theta} = x$, the radial and tangential component velocity ratios of the sliding block, so that the angular momentum in terms of the velocity ratios is:

$$\left[\left(m + m_c\frac{h}{l}\right)xK_{\lambda\theta} + (I - m_c hl)K_{\psi\theta} + I_f\right]\dot{\theta}.$$

For the rate of change of the angular momentum about the shaft axis, R_f being the frame reaction, we have:

$$R_f - R = \left[\left(m + m_c\frac{h}{l}\right)x^2 + I_f\right]\ddot{\theta} + (I - m_c hl)\ddot{\psi} + 2\left(m + m_c\frac{h}{l}\right)x\dot{x}\dot{\theta},$$

or expressed in terms of the velocity ratios:

$$R_f - R = \left[\left(m + m_c\frac{h}{l}\right)xK_{\lambda\theta} + (I - m_c hl)K_{\psi\theta} + I_f\right]\ddot{\theta} + \left[\left(m + m_c\frac{h}{l}\right)\left(x\frac{dK_{\lambda\theta}}{d\theta} + K_{\lambda\theta}\frac{dx}{d\theta}\right) + (I - m_c hl)\frac{dK_{\psi\theta}}{d\theta}\right]\dot{\theta}^2.$$

Since

$$\ddot{\psi} = K_\psi\ddot{\theta} + \frac{dK_{\psi\theta}}{d\theta}\dot{\theta}^2$$

and

$$K_{x\theta} = x, \qquad \frac{dx}{d\theta} = K_{x\theta} = \frac{dK_{x\theta}}{d\theta},$$

we may write from either expression,

$$R_f - R = \left[\left(m + m_c\frac{h}{l}\right)x K_{x\theta} + (I - m_c hl)K_{\phi\theta} + I_f\right]\ddot{\theta} + \left[2\left(m + m_c\frac{h}{l}\right)x\frac{dK_{x\theta}}{d\theta} + (I - m_c hl)\frac{dK_{\phi\theta}}{d\theta}\right]\dot{\theta}^2,$$

where we note $\frac{dK_{x\theta}}{d\theta} = K_{x\theta}$ and $K_{x\theta} = x$.

Rotating Frame Swinging Block Linkage (Fig. 38).—Consider the rotating frame or link AFE rotating about axis A, with sliding block C connected the lever BC to axis B (Fig. 38).

Let I_f = moment of inertia of AFE about A,
I = moment of inertia of BC about B,
m_c = mass of sliding block C,
θ = angular displacement of AFE with respect to the fixed frame AD,
ϕ = angular displacement of link BC,
x, y = instantaneous coördinate axis of the sliding block normal and along the rotating frame,
r = length of link BC.

The kinetic energy of the system is:

$$T = \tfrac{1}{2}I_f\dot{\theta}^2 + \tfrac{1}{2}I\dot{\phi}^2 + \tfrac{1}{2}m_c[(\dot{y} + x_0\dot{\theta})^2 + y^2\dot{\theta}^2],$$

where it is to be noted:

$$x_0 = AF\cos\alpha, \text{ a constant,}$$

and

$$(\dot{y} + x_0\dot{\theta})^2 + y^2\dot{\theta}^2 = r^2\dot{\phi}^2.$$

For the kinematic conditions, we have:

$$\delta x = -y\dot{\theta}\delta t,$$
$$r\dot{\phi}\cos(\theta + \phi) = y\dot{\theta},$$
$$r\dot{\phi}\sin(\theta + \phi) = \dot{y} + x_0\dot{\theta},$$

so that the velocity ratios and their derivatives are:

$$K_{\phi\theta} = \frac{y}{r\cos(\phi+\theta)}; \qquad K_{y\theta} = y\tan(\phi+\theta) - x_0$$

and

$$K_{x\theta} = \frac{\dot{x}}{\dot{\theta}} = -y,$$

$$\frac{dK_{\phi\theta}}{d\theta} = \frac{K_{y\theta}}{r}\sec(\theta+\phi) + \frac{y}{r}(1+K_{\phi\theta})\tan(\phi+\theta)\sec(\phi+\theta),$$

$$\frac{dK_{y\theta}}{d\theta} = K_{y\theta}\tan(\phi+\theta) + y(1+K_{\phi\theta})\sec^2(\phi+\theta).$$

The kinetic energy in terms of the velocity ratios and the fundamental coördinate θ is:

$$T = \tfrac{1}{2}[I_f + IK_{\phi\theta}^2 + m_c(K_{y\theta}^2 + 2K_{y\theta}x_0 + x_0^2 + y^2)]\dot{\theta}^2,$$

so that the coefficient of inertia of the system is:

$$A = I_f + IK_{\phi\theta}^2 + m_c(K_{y\theta}^2 + 2xK_{y\theta} + (x_0^2 + y^2)).$$

If R_θ is the torque on the shaft A and R_ϕ the resisting torque on the shaft B, then:

$$R_\theta + K_{\phi\theta}R_\phi = A\ddot{\theta} + \left[IK_{\phi\theta}\frac{dK_{\phi\theta}}{d\theta} + m_c\left(K_{y\theta}\frac{dK_{y\theta}}{d\theta} + x_0\frac{dK_{y\theta}}{d\theta} + yK_{y\theta}\right)\right]\dot{\theta}^2.$$

Since the block participates in the motion of the point C of the lever BC, we may also write:

$$R_\theta + K_{\phi\theta}R_\phi = [I_f + (I + m_cr^2)K_{\phi\theta}^2]\ddot{\theta} + \left[(I + m_cr^2)K_{\phi\theta}\frac{dK_{\phi\theta}}{d\theta}\right]\dot{\theta}^2,$$

since

$$r^2K_{\phi\theta}^2 = K_{y\theta}^2 + 2K_{y\theta}x_0 + x_0^2 + y^2$$

and

$$r^2K_{\phi\theta}\frac{dK_{\phi\theta}}{d\theta} = K_{y\theta}\frac{dK_{y\theta}}{d\theta} + yK_{y\theta} + x_0\frac{dK_{y\theta}}{d\theta}.$$

By Direct Analysis or the Synthetic Method (Fig. 38).—Let N = normal reaction between block C and the rotating frame AFE,

X_c and Y_c = the components normal and along the slide FE of the hinge-joint reaction between the block C and the lever BC at C.

Then for the rotating frame:

$$R_\theta - Ny = I_f\ddot{\theta}$$

and for the sliding block C:

$$N - X_c = m_c(y\ddot{\theta} + x_0\dot{\theta}^2 + 2\dot{y}\dot{\theta}),$$
$$Y_c = m_c(\ddot{y} + x_0\ddot{\theta} - y\dot{\theta}^2),$$

and for the motion of the lever BC:

$$R_\phi + X_c r\cos(\phi + \theta) - Y_c r\sin(\phi + \theta)\, I = .\ddot{\phi}$$

Eliminating N and X_c and Y_c, we have:

$$R_\theta + K_{\phi\theta}\cdot R_\phi - I\ddot{\phi}K_{\phi\theta} - I_f\ddot{\theta} - m_c(\ddot{y} + x_0\ddot{\theta} - y\dot{\theta}^2)(K_{y\theta} + x_0) - m_c(y\ddot{\theta} + x_0\dot{\theta}^2 + 2\dot{y}\dot{\theta})y = 0,$$

which is the equation of the kinetic reactions reduced to the coördinate θ.

The acceleration may be expressed in terms of the velocity ratios and their derivatives:

$$\ddot{\phi} = K_{\phi\theta}\ddot{\theta} + \frac{dK_{\phi\theta}}{d\theta}\dot{\theta}^2,$$

$$\ddot{y} = K_{y\theta}\ddot{\theta} + \frac{dK_{y\theta}}{d\theta}\dot{\theta}^2,$$

so that on substituting, we have:

$$R_\theta + K_{\phi\theta}R_\phi = [I_f + IK_{\phi\theta}^2 + m_c(K_{y\theta}^2 + 2x_0K_{y\theta} + x^2 + y^2)]\ddot{\theta} + \left[IK_{\phi\theta}\frac{dK_{\phi\theta}}{d\theta} + m_c\left(K_{y\theta}\frac{dK_{y\theta}}{d\theta} + yK_{y\theta} + x_0\frac{dK_{y\theta}}{d\theta}\right)\right]\dot{\theta}^2,$$

which verifies the previous derivation.

Frame Reaction and Rate of Change of the Angular Momentum of System (Fig. 38).—The external reactions consists of the resultant bearing reactions of the shaft divided

into components, Y_A and X_A at A and Y_B and X_B at B, together with the torque reactions on the shafts R_θ at A and R_ϕ at B.

Then taking moments about the shaft A, we have (Fig. 38): $R_\theta - R_\phi + Y_B m + X_B h$ = rate of change of angular momentum about A. Since the torque reaction on the frame is:

$$-R_f = -(Y_B m + X_B h).$$

Therefore,

$$R_f + R_\theta - R_\phi = I_f\ddot{\theta} + \left(m_c + m_{BC}\frac{z_B}{r}\right)[(y\ddot{\theta} + x_0\dot{\theta}^2 + 2\dot{y}\dot{\theta})y + (\ddot{y} + x_0\ddot{\theta} - y\dot{\theta}^2)x_0] + (I - m_{BC}z_B r)\ddot{\phi}.$$

Since

$$\dot{y} = K_{y\theta}\dot{\theta}, \qquad \ddot{y} = K_{y\theta}\ddot{\theta} + \frac{dK_{y\theta}}{d\theta}\dot{\theta}^2, \qquad \ddot{\phi} = K_{\phi\theta}\ddot{\theta} + \frac{dK_{\phi\theta}}{d\theta}\dot{\theta}^2.$$

The torque reaction of the frame in terms of the velocity ratios and the primary coördinate is:

$$R_f = \left[I_f + (I - m_{BC}z_B r)K_{\phi\theta} + \left(m_c + m_{BC}\frac{z_B}{r}\right)(y^2 + x_0^2 + x_0 K_{y\theta})\right]\ddot{\theta} + \left[(I - m_{BC}z_B r)\frac{dK_{\phi\theta}}{d\theta} + \left(m_c + m_{BC}\frac{z_B}{r}\right) \times \left(x_0\frac{dK_{y\theta}}{d\theta} + 2yK_{y\theta}\right)\right]\dot{\theta}^2 - R_\theta + R_\phi,$$

where, if R_θ and R_ϕ are given, $\ddot{\theta}$ is determined from the generalized equation of motion.

Oscillating Engine Mechanism (Fig. 39).—The velocity ratios are:

$$r\dot{\theta}\sin(\theta + \phi) = \dot{z}, \qquad K_{z\theta} = r\sin(\theta + \phi),$$

$$r\dot{\theta}\cos(\theta + \phi) = z_c\dot{\phi}, \qquad K_{\phi\theta} = \frac{r}{z_c}\cos(\theta + \phi),$$

$\frac{dK_{z\theta}}{d\theta} = r\cos(\theta + \phi)$ since ϕ is constant for a resolution along the inclined direction of the rod,

$$\frac{dK_{\phi\theta}}{d\theta} = -\frac{r}{z_c}\sin(\theta+\phi)(1+K_{\phi\theta}) - \frac{r}{z_c^2}\cos(\theta+\phi)K_{z\theta},$$

since z_c is a variable, and $\frac{dz_c}{d\theta} = \frac{dz}{d\theta} = K_{z\theta}$.

Let I_c = moment of inertia of cylinder about trunnions,

I_r = moment of inertia of rod and piston about crank pin center,

m_r = mass of rod and piston,

z_g = distance of c. of g. of rod and piston from crank pin.

Then the kinetic energy of the system is:

$$T = \tfrac{1}{2}I_c\dot{\phi}^2 + \tfrac{1}{2}I_r\dot{\phi}^2 + \tfrac{1}{2}m_r[r^2\dot{\theta}^2 - 2z_g r\dot{\theta}\dot{\phi}\cos(\theta+\phi)],$$

so that the equivalent mass in terms of the velocity ratios is:

$$A = (I_c + I_r)K_{\phi\theta}^2 + m_r(r^2 - 2z_g r K_{\phi\theta}\cos(\theta+\phi)).$$

The variation of the equivalent mass with respect to the coördinate θ is:

$$\frac{dA}{d\theta} = 2(I_c+I_r)K_{\phi\theta}\frac{dK_{\phi\theta}}{d\theta} - 2m_r z_g r\frac{dK_{\phi\theta}}{d\theta}\cos(\theta+\phi) + 2m_r z_g r K_{\phi\theta}\sin(\theta+\phi)(1+K_{\phi\theta}).$$

The work of the internal fluid pressure in the cylinder reduced to the coördinate θ is:

$$P\frac{dx}{d\theta}\cdot d\theta, \text{ so that } P_\theta = PK_{x\theta} = Pr\sin(\theta+\phi).$$

If R_θ is the resisting torque on the shaft, the equation of motion of the system is:

$$PK_x - R_\theta = [(I_c+I_r)K_{\phi\theta}^2 + m_r(r^2 - 2z_g r K_{\phi\theta}\cos(\theta+\phi))]\ddot{\theta} + \left[(I_c+I_r)K_{\phi\theta} - m_r z_g r\cos(\theta+\phi)\frac{dK_{\phi\theta}}{d\theta} + m_r z_g r K_{\phi\theta}\sin(\theta+\phi)(1+K_{\phi\theta})\right]\dot{\theta}^2.$$

Substituting the velocity ratios and their derivatives and simplifying, we have:

$$Pr \sin(\theta + \phi) - R_g = \left[(I_c + I_r)\left(\frac{r}{z_c}\right)^2 \cos^2(\theta + \phi) + m_r\left(r^2 - 2z_g \frac{r^2}{z_c} \cos^2(\theta + \phi)\right)\right]\ddot{\theta} - \left[((I_c + I_r) - 2m_r z_g z_c)\left(\frac{r}{z_c}\right)^2 \frac{\sin 2(\theta + \phi)}{2} \times \left(1 + \frac{r}{z_c}\cos(\theta + \phi)\right) + ((I_c + I_r) - m_r z_g z_c)\left(\frac{r}{z_c}\right)^3 \times \frac{\sin 2(\theta + \phi)}{2} \cos(\theta + \phi)\right]\dot{\theta}^2,$$

which is the equation of motion of the system in terms of the coördinate of the crank θ.

Calculation of Torque by Synthetic Procedure (Fig. 40).—We may readily verify the previous analysis by a direct calculation of the reactions and their moment about the shaft:

Considering the rod and piston with the cylinder collectively, we have, for moments about the trunnion (Fig. (*a*)):

$$Y_B z_c = (I_c + I_{rg})\ddot{\phi} + m_r(z_c - z_g)^2\ddot{\phi} + 2m_r \dot{z}_r \dot{\phi}(z_c - z_g) \quad (1)$$

or alternatively,

$$Y_B z_c = (I_c + I_{rg})\ddot{\phi} + m_r(r\ddot{\theta}\cos(\theta + \phi) - r\dot{\theta}^2 \sin(\theta + \phi)) \times (z_c - z_g) - m_r z_g (z_c - z_g)\ddot{\phi}.$$

Now considering, Fig. (*b*) showing the reactions on the cylinder, evidently,

$$X_c = P$$

so that

$$X_B = P + m_r(z_c - z_g)\dot{\phi}^2 - m_r \ddot{z}_r \quad (2)$$

or alternately,

$$X_B = P - m_r(r\ddot{\theta}\sin(\theta + \phi) + r\dot{\theta}^2\cos(\theta + \phi)) - m_r z_g \dot{\phi}^2. \quad (2')$$

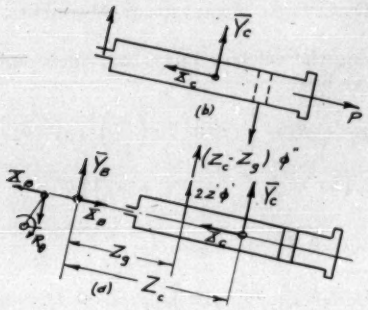


FIG 40

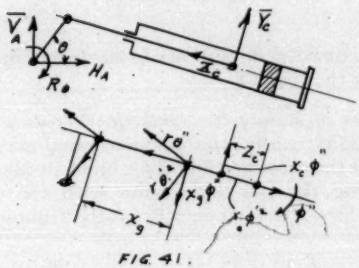


FIG. 41.

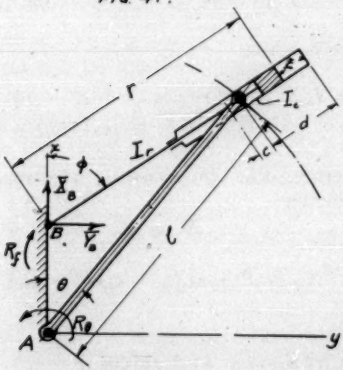


FIG. 42.

The equation for the torque is:

$$R_\theta = X_B r \sin(\theta + \phi) - Y_B r \cos(\theta + \phi). \qquad (3)$$

Substituting (1) and (2) in (3), we have:

$$\begin{aligned} R_\theta = {} & Pr \sin(\theta + \phi) + m_r r(z_c - z_g) \sin(\theta + \phi)\dot{\phi}^2 \\ & - m_r r \sin(\theta + \phi)\ddot{z}_r - (I_c + I_{rg})\frac{r}{z_c}\cos(\theta + \phi)\cdot\ddot{\phi} \\ & - m_r(z_c - z_g)^2 \frac{r}{z_c}\cos(\theta + \phi)\ddot{\phi} \\ & - 2m_r r\dot{z}_r\dot{\phi}\left(\frac{z_c - z_g}{z_a}\right)\cos(\theta + \phi) \end{aligned}$$

or alternately,

$$\begin{aligned} R_\theta = {} & Pr \sin(\theta + \phi) \\ & - m_r(r\ddot{\theta}\sin(\theta + \phi) + r\dot{\theta}^2\cos(\theta + \phi))r\sin(\theta + \phi) \\ & - m_r r z_g \sin(\theta + \phi)\dot{\phi}^2 - (I_c + I_{rg})\frac{r}{z_c}\cos(\theta + \phi)\ddot{\phi} \\ & - m_r\frac{r}{z_c}(r\ddot{\theta}\cos(\theta + \phi) - r\dot{\theta}^2\sin(\theta + \phi))\cos(\theta + \phi) \\ & + m_r z_g \frac{r}{z_c}\cos(\theta + \phi)\ddot{\phi}, \end{aligned}$$

where the velocities and accelerations are:

$$\dot{\phi} = \frac{r}{z_c}\dot{\theta}\cos(\theta + \phi),$$

$$\ddot{\phi} = \frac{r}{z_c}\ddot{\theta}\cos(\theta + \phi) - \frac{r}{z_c}\dot{\theta}^2\sin(\theta + \phi)\left(1 + 2\frac{r}{z_c}\cos(\theta + \phi)\right),$$

$$\dot{z}_r = r\dot{\theta}\sin(\theta + \phi),$$

$$\ddot{z}_r = r\ddot{\theta}\sin(\theta + \phi) + r\dot{\theta}^2\cos(\theta + \phi)\left(1 + \frac{r}{z_c}\cos(\theta + \phi)\right).$$

Substituting for these values in either expression for the torque, we have:

$$R_\theta = Pr\sin(\theta + \phi) - \left[(I_c + I_r)\left(\frac{r}{z_0}\right)^2 \cdot \cos^2(\theta + \phi)\right.$$

$$+ m_r\left(r^2 - 2z_g\frac{r^2}{z_c}\cos^2(\theta+\phi)\right)\Big]\ddot{\theta}$$

$$+\Big[((I_c + I_r) - 2m_r z_g z_c)\left(\frac{r}{z_c}\right)^2$$

$$\times\frac{\sin 2(\theta+\phi)}{2}\left(1 + \frac{r}{z_c}\cos(\theta+\phi)\right)$$

$$-((I_c + I_r) - m_r z_g z_c)\left(\frac{r}{z_c}\right)^3$$

$$\times\frac{\sin 2(\theta+\phi)}{2}\cos(\theta+\phi)\Big]\dot{\theta}^2,$$

which verifies the previous analysis obtained by a direct application of the general dynamical equation in terms of the velocity ratios and their derivatives.

Torque Reaction on Frame (Fig. 41).—If R_f is the torque reaction of the frame about the shaft, R_θ the resisting torque on the shaft, then, $R_f - R_\theta$ = rate of change of the angular momentum of the system about the axis of shaft $= \frac{dL_\theta}{dt}$.

Evidently $\frac{dL_\theta}{dt}$ is the moment of the kinetic reaction of the members about the axis of the shaft, and R_f is the moment of the trunnion reaction about the shaft, that is:

$$R_f = (Y_c\cos\phi + X_c\sin\phi)(z_c\cos\phi + r\cos\theta).$$

The rate of change of angular momentum is,

$$\frac{dL_\theta}{dt} = m_r[(r + z_g\cos(\theta+\phi)\cdot r\ddot{\theta} + z_g r\sin(\theta+\phi)(\dot{\phi}^2 - \dot{\theta}^2)$$

$$- rz_g\cos(\theta+\phi)\ddot{\phi}] - (I_r + I_c)\ddot{\phi}$$

$$+ m_c z_{cg}[(r\cos(\theta+\phi) + z_c)\ddot{\phi} - r\sin(\theta+\phi)\dot{\phi}^2],$$

where z_c = distance of c. of g. of cylinder from trunnion

$$\ddot{\phi} = K_{\phi\theta}\ddot{\theta} + \frac{dK_{\phi\theta}}{d\theta}\dot{\theta}^2, \qquad \dot{\phi} = \frac{r}{z_c}\cos(\phi+\theta)\dot{\theta};$$

$$\therefore\quad \ddot{\phi} = \frac{r}{z_c}\cos(\theta+\phi)\ddot{\theta} - \frac{r}{z_c}\left(\sin(\theta+\phi) + \frac{r}{z_c}\sin 2(\theta+\phi)\right)\dot{\theta}^2$$

Dynamics of the Rotating Frame Oscillating Engine (Fig. 42).—Let A be the shaft axis, B a fixed axis attached to the frame about which the piston rod rotates. Let the length of the torque lever keyed to the shaft A and connecting with the trunnions of the oscillating cylinder be l. It is to be noted with several cylinders the torque levers will be connected up into a revolving frame. The overall length of the piston rod is r. The distance from the trunnions to the outward end of the cylinder is d, while the relative displacement of the piston from the outward cylinder end is ξ.

Let (Fig. 42) ξ = the relative displacement of piston in the cylinder.

θ_1 and ϕ = The shaft angle and the piston rod angular displacement with respect to a fixed line in the frame.

ψ = The angle between the piston rod and torque lever.

I_l = moment of inertia of torque lever about the shaft axis.

I_r = The moment of inertia of the piston rod about axis B.

I_c = moment of inertia of oscillating cylinder about the trunnions.

m_c = mass of oscillating cylinder, where the c. of g. is located at distance C, inward from trunnion.

The general kinematic relations are (Fig. 42):

$$l\theta \cos \psi = (r + \xi - d)\phi,$$
$$\dot{\xi} = l\theta \sin \psi, \qquad \phi = \psi + \theta,$$

so that the velocity ratios with respect to the shaft are,

$$K_{\phi\theta} = \frac{l \cos \psi}{r + \xi - d},$$

$$K_{\psi\theta} = K_{\phi\theta} - 1 = \frac{l \cos \psi - r - \xi + d}{r + \xi - d},$$

$$K_{\xi\theta} = l \sin \psi = h \sin \phi,$$

while their derivatives are

$$\frac{dK_{\phi\theta}}{d\theta} = -\frac{l\sin\psi}{r+\xi-d}\cdot K_{\psi\theta} - \frac{l\cos\psi}{(r+\xi-d)^2}\cdot K_{\xi\theta},$$

$$\frac{dK_{\psi\theta}}{d\theta} = \frac{dK_{\phi\theta}}{d\theta} = -\frac{l}{r+\xi-d}\left(K_{\psi\theta}\sin\psi + \frac{K_{\xi\theta}\cos\psi}{r+\xi-d}\right)$$

$$\frac{dK_{\xi\theta}}{d\theta} = l\cos\psi\cdot K_{\psi\theta} = h\cos\phi\cdot K_{\phi\theta},$$

the kinetic energy of the mechanism is

$$T = \tfrac{1}{2}[I_l\dot{\theta}^2 + (I_r+I_c)\dot{\phi}^2 + m_c(l^2\dot{\theta}^2 - 2lc\dot{\theta}\dot{\phi}\cos\psi)$$
$$= \tfrac{1}{2}[I_l + (I_r+I_c)K_{\phi\theta}^2 + m_c(l^2 - 2lc\cos\psi\cdot K_{\phi\theta})]\dot{\theta}^2,$$

so that the coefficient of inertia in terms of the velocity ratio is

$$A = I_l + (I_r+I_c)K_{\phi\theta}^2 + m_c(l^2 - 2lc\cos\psi\cdot K_{\phi\theta})$$

and the variation of the inertia coefficient with respect to the shaft coördinate is

$$\frac{1}{2}\frac{dA}{d\theta} = (I_r+I_c)K_{\phi\theta}\frac{dK_{\phi\theta}}{d\theta} + m_c lc\left(\sin\psi\cdot K_\theta K_{\phi\theta} - \cos\psi\frac{dK_{\phi\theta}}{d\theta}\right).$$

The reduced force of the gas pressures in the cylinder is

$$P\frac{d\xi}{d\theta} = PK_{\xi\theta} = Pl\sin\psi = Ph\sin\phi.$$

The equation for the torque on the propeller shaft R_θ is therefore

$$[I_l + (I_r+I_c)K_{\phi\theta}^2 + m_c(l^2 - 2lc\cos\psi\cdot K_{\phi\theta})]\ddot{\theta}$$
$$+\left[(I_r+I_c)K_{\phi\theta}\frac{dK_{\phi\theta}}{d\theta} + m_c lc\left(\sin\psi\cdot K_{\psi\theta}\cdot K_{\phi\theta} - \cos\psi\frac{dK_{\phi\theta}}{d\theta}\right)\right]\dot{\theta}^2 = PK_{\xi\theta} - R_\theta.$$

Torque reaction $-R_f$ on engine frame (Fig. 42):

Let r_g = distance from B to c. of g. of piston rod,

m_r = mass of piston and piston,

I_{rg} = moment of inertia of piston rod and piston about its c. of g.,

Y_B, X_B = the reactions at B.

Then we notice

$$I_r = I_{rg} + m_r \cdot r_g^2 \quad \text{and} \quad R_f = Y_B \cdot h.$$

The rate of change of angular momentum of the system about A is equal to the moment of the kinetic reactions, so that,

$$R_f - R_\theta = [I_l + m_c l(l - c\cos\psi)]\ddot{\theta} - m_c lc \sin\psi \cdot \dot{\theta}^2 + [I_r + m_c c(c - l\cos\psi)]\ddot{\phi} + m_c lc \sin\psi \cdot \dot{\phi}^2.$$

The internal gas pressure reacts equally and opposite on the cylinder head and piston and therefore its moment effect is nil.

Further, since

$$\ddot{\phi} = K_{\phi\theta}\ddot{\theta} + \frac{dK_{\phi\theta}}{d\theta} \cdot \dot{\theta}^2, \qquad \dot{\phi}^2 = K_{\phi\theta}^2 \dot{\theta}^2.$$

The torque reaction on the frame $-R_f$ reduced to the primary coördinate is, therefore,

$$R_f = R_\theta + [I_l + I_r K_{\phi\theta} + m_c(l(l - c\cos\psi) + cK_{\phi\theta}(c - l\cos\psi))]\ddot{\theta} + \left[(I_r + m_c c(c - l\cos\psi))\frac{dK_{\phi\theta}}{d\theta} + m_c lc \sin\psi(K_{\phi\theta}^2 - 1)\right]\dot{\theta}^2.$$

Dynamics of the Reciprocating Engine (Fig. 43).—The reciprocating engine mechanism (sliding bar linkage) offers a simple expression in terms of the coördinate of the crank angle. If we know the nature of the applied force as the steam pressure and the resisting moment at the shaft we may determine the oscillation and maximum acceleration of the system as a whole.

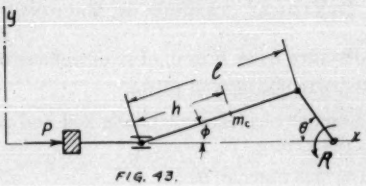


FIG. 43.

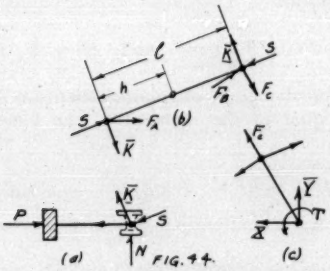


FIG. 44.

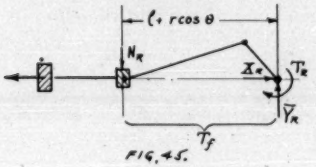


FIG. 45.

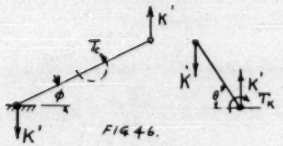


FIG 46.

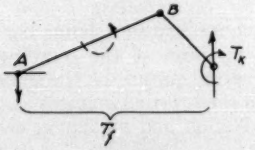


FIG. 47.

Let m_r = mass of reciprocating parts,

$m_0 k_0^2$ = the moment of inertia of the crank and connected shafting,

m_c = mass of connecting rod,

h = distance from crosshead to c. of g. of connecting rod,

k = radius of gyration of rod about crosshead,

r = crank radius,

ϕ = angle of obliquity of connecting rod,

P = piston load due to steam pressure,

R = resisting torque on driving shaft.

The kinetic energy of the mechanism is:

$$T = \frac{1}{2}\left[\left(m_r + m_c\left(\frac{l-h}{l}\right)\right)\dot{x}^2 + \left(m_0 k_0^2 + m_c \frac{h}{l} r^2\right)\dot{\theta}^2 + m_c(k^2 - hl)\dot{\phi}^2\right].$$

The velocity ratios are:

$$K_{x\theta} = r\left(\sin\theta + \frac{r}{2l}\sin 2\theta\right),$$

$$\frac{dK_{x\theta}}{d\theta} = r\left(\cos\theta + \frac{r}{l}\cos 2\theta\right),$$

$$K_{\phi\theta} = \frac{r}{l}\cos\theta \quad \text{and} \quad \frac{dK_{\phi\theta}}{d\theta} = -\frac{r}{l}\sin\theta.$$

The coefficient of inertia of the system in terms of the velocity ratios is, therefore,

$$A = \left(m_r + m_c\frac{l-h}{l}\right)K_{x\theta}^2 + m_0 k_0^2 + m_c\frac{h}{l}r^2 + m_c(k^2 - hl)K_{\phi\theta}^2.$$

The generalized equation of motion is, therefore,

$$\left[\left(m_r + m_c\frac{l-h}{l}\right)K_{x\theta}^2 + m_0 k_0^2 + m_c\frac{h}{l}r^2 + m_c(k^2 - hl)K_{\phi\theta}^2\right]\ddot{\theta} +$$

$$\left[\left(m_r + m_c\frac{l-h}{l}\right)K_{x\theta}\frac{dK_{x\theta}}{d\theta} + m_c(k^2 - hl)K_{\phi\theta}\frac{dK_{\phi\theta}}{d\theta}\right]\dot{\theta}^2 = PK_{x\theta} - R.$$

If we let

$$m_r' = m_r + m_c\frac{l-h}{l}, \qquad m_0'k_0^2 = m_0k_0^2 + m_c\frac{h}{l}r^2,$$
$$m_ck_1^2 = m_c(k^2 - hl)$$

and then substitute the velocity ratios, the generalized equation for the reciprocating engine is:

$$Pr\left(\sin\theta + \frac{r}{2l}\sin 2\theta\right) - R = \left[m_r'r^2\left(\sin\theta + \frac{r}{2l}\sin 2\theta\right)^2 + m_ck_1^2\frac{r^2}{l^2}\cos^2\theta + m_0'k_0^2\right]\ddot{\theta} + \left[m_r'r^2\left(\sin\theta + \frac{r}{2l}\sin 2\theta\right)\left(\cos\theta + \frac{r}{l}\cos 2\theta\right) - m_ck_1^2\cdot\frac{r^2}{2l^2}\sin 2\theta\right]\dot{\theta}^2.$$

These equations may be verified by a direct analysis (Fig. 44).

The reciprocating parts (a), connecting rod (b), and crank and shaft (c) are separated showing the reactions on each part.

The connecting rod is subjected to the following components:

1. A common thrust S with equal and opposite component reactions at A and B.

2. A reaction $F_A = m_c\left(\frac{l-h}{l}\right)\ddot{x} = m_{cA}\ddot{x}$ at A.

3. A couple $K = m_c(k^2 - hl)\ddot{\phi}$ with equal and opposite components normal to the rod at A and B.

4. A centrifugal and tangential component at B

$$F_c = m_c\frac{h}{l}\dot{\theta}^2r, \qquad F_b = m_c\frac{h}{l}r\ddot{\theta} = m_{cB}r\ddot{\theta}.$$

For the reciprocating parts, we have:

$$P - S\cos\phi - K\sin B - m_{cA}\ddot{x} = m_R\ddot{x};$$
$$\therefore\ S = \frac{P - K\sin\phi - m_R'\ddot{x}'}{\cos\phi}, \qquad m_R' = m_R + m_{cA}.$$

Now considering the moments on the crank and shaft, we have:

$$Sr \sin(\theta + \phi) - Kr \cos(\theta + \phi) - F_B r - R = m_0 k_0^2 \ddot{\theta}.$$

Now

$$\frac{\sin(\theta + \phi)}{\cos \phi} = \sin \theta + \frac{r}{2l} \sin 2\theta \text{ (approx.)};$$

$$\therefore \quad (P - K \sin \phi - m_R' \ddot{x}) \left(\sin \theta + \frac{r}{2l} \sin 2\theta \right) r$$

$$- Kr(\cos \theta \cos \phi - \sin \theta \sin \phi) - m_{cB} r^2 \ddot{\theta} - R = m_0 k_0^2 \ddot{\theta}.$$

Substituting the previous kinematic values for x and ϕ and noting $K = m_c \frac{(k^2 - hl)}{l} = m_c \frac{k'^2}{l}$, we have:

$$Pr\left(\sin \theta + \frac{r}{2l} \sin 2\theta \right) - R = \left[m_R' r^2 \left(\sin \theta + \frac{r}{2l} \sin 2\theta \right)^2 \right.$$

$$\left. + m_c k'^2 \frac{r^2}{l^2} \cos^2 \theta + m_0' k_0'^2 \right] \ddot{\theta}$$

$$+ \left[m_R' r^2 \left(\sin \theta + \frac{r}{2l} \sin 2\theta \right) \left(\cos \theta + \frac{r}{l} \cos 2\theta \right) \right.$$

$$\left. - m_c k'^2 \frac{r^2}{2l^2} \sin 2\theta \right] \dot{\theta}^2,$$

where terms over the second power of r/l are omitted and assuming $\cos \phi = 1$ (approx.). This verifies with the same degree of approximation the previous dynamical equation.

(To be continued)

The Electrical Resistance of Thin Films of Metals. W. BRAUNBEK. (*Zeit. f. Physik*, Vol. 59, Nos. 3 and 4.) In 1921 Volmer and Estermann published a study of the evaporation of mercury at temperatures ranging from 59° C. to − 64° C. For temperatures from 0° to − 64° C. a horizontal surface of mercury maintained at a definite temperature, was placed 3 mm. below a parallel glass surface cooled with liquid air. The intervening space was a vacuum. Close beneath the surface of glass was a screen with an aperture 10 mm. long and 1 mm. wide. The metallic vapor passing upwards from the mercury in part condensed on the screen, but in addition formed a rectangular patch on the glass, the thickness of which increased with time. The electrical resistance of this patch of solid mercury was measured, and its rate of deposit was thereby examined. In the present paper are worked over for data on the resistance of thin films the results originally obtained in order to calculate the evaporation of mercury.

The specific resistance of mercury in thin films varies with the thickness of the film very much as the resistance of other metals changes. "The specific resistance is very great for very thin films, with increasing thickness at first diminishes rapidly, then more slowly and plainly tends asymptotically to a limiting value for thick films." The following table illustrates these relations:

Thickness in millionths of a millimeter	220	430	620	1060	2420
Specific Resistance in 10^{-6} ohms.	22	4.3	3.1	2.65	2.42

The thickest film here listed had a specific resistance about three times as great as that of solid mercury in bulk at the temperature of liquid air, as found by Kamerlingh Onnes.

G. F. S.

COMMITTEE ON SCIENCE AND THE ARTS.

ABSTRACTS OF REPORTS.

THE KLYDONOGRAPH.

HALL OF THE INSTITUTE,
PHILADELPHIA, February 6, 1929.

No. 2840.

The Franklin Institute of the State of Pennsylvania, acting through its Committee on Science and the Arts, investigating the Klydonograph, invented by Mr. J. F. Peters of Swissvale, Pennsylvania, reports as follows:

The Klydonograph which is the subject of this investigation may be called a wave writer since it is used for obtaining detailed information concerning the surges that take place on extended transmission lines. This information has to do with the polarity, magnitude, steepness of the wave front, etc.; the instrument makes a permanent record by means of which these details can be interpreted.

The instrument is based upon the figures that are obtained when a condenser discharge takes place across a spark gap onto a terminal in contact with an insulating plate resting upon a grounded plate. Whenever a powder is sifted upon the upper plate and a spark is passed, figures are outlined in the powder and show differences depending upon the character of the discharge.

This is a well-known demonstration in physical laboratories and since it was first described by Doctor Lichtenburg in 1777 the results have been known as "Lichtenburg figures."

It was early found that certain powders of different colors would give beautiful colored figures and it was known that the figure produced by a positive discharge was notably different from that produced by a negative discharge.

If a photographic dry plate was used it was found that, on supporting the glass side of the plate on a metal plate connected to the earth and letting the discharge strike the film side of the plate, the paths of the discharge over the film would give characteristic figures when the film was developed.

These facts have been known for years and the work done by Mr. J. F. Peters of the Transformer Engineering Department of the Westinghouse Electric and Manufacturing Company has been that of standardizing the form of instrument used, controlling the conditions under which the figures are obtained, making a comparative study of these figures and interpreting the meaning of their forms in the light of the known condition.

Figure *1* of Plate I shows in outline a simple form of instru-

PLATE I

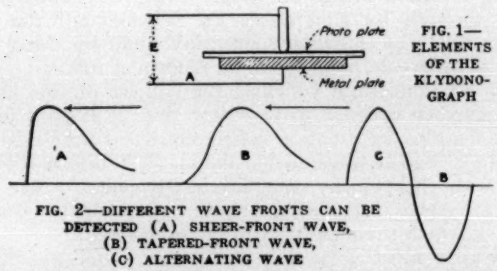


FIG. 1—ELEMENTS OF THE KLYDONOGRAPH

FIG. 2—DIFFERENT WAVE FRONTS CAN BE DETECTED (A) SHEER-FRONT WAVE, (B) TAPERED-FRONT WAVE, (C) ALTERNATING WAVE

ment which includes the essential elements of the klydonograph with the discharge pole resting on the film surface of the photographic plate. Figure *2* shows a sketch of different wave fronts such as may be found in surges on transmission lines. The polarity of the surge voltage is indicated by the appearance of the figure while an approximate idea of the steepness of the wave front may be had as is shown in Figures *A*, *B* and *C*. If these surges are in one direction only, the figure formed on the photographic plate will indicate whether the surge was of positive or negative polarity and will also tell whether the wave front was of the abrupt type shown at *A*, Figure *2* or of the sloping front shown at *B*.

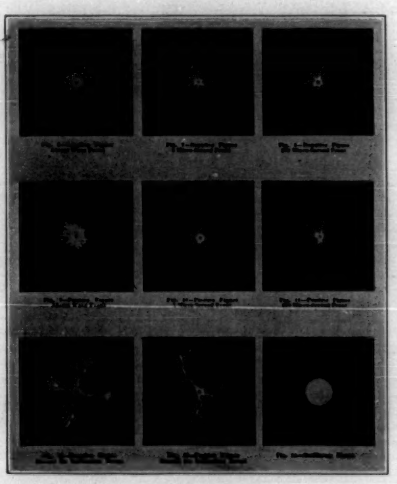
The length of line covered by the slope of a surge is indicated by taking the product of the velocity of transmission by the time it takes for the potential to rise from zero to a maximum. Assuming the velocity of transmission to be that of light the length of line covered by a 5 microsecond

slope would be 0.93 mile, or approximately one mile. That for a 200 microsecond slope would be approximately forty miles.

If the surge is alternating, as at *C*, the resulting figure will disclose the fact.

If the curve of *B*, Figure *2*, is drawn to scale the axis of *X* would represent time and *Y* voltage. To reach a maximum voltage for the surge representing a sloping front of one mile

PLATE II



in length would require five millionths of a second. This is called a wave with a 5 microsecond front.

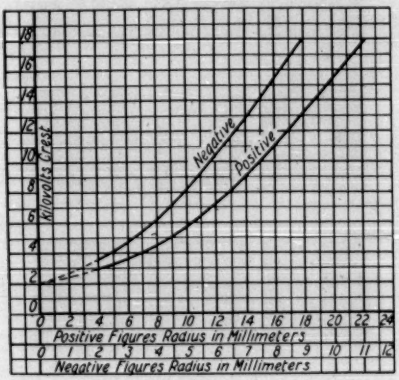
Figures *6* to *14* of Plate II are reproductions of prints from negatives that were made with a simple type of klydonograph. Figures *6* and *9* show the difference in figures produced by negative and positive surges both having a steep front. Figures *7* and *10* show the difference obtained from negative and positive surges each having a 5 microsecond front. Figures *8* and *11* compare the figures obtained from negative

and positive surges having a 200 microsecond front. Figure *12* shows the figure obtained from a negative surge that is beyond the calibration range and Figure *13* shows the figure from a similar positive surge. Figure *14* is the result of an alternating discharge. When high voltage surges take place there is liable to be a spark across the plate with the result that the plate is fogged.

The magnitude of the surge voltage on the klydonograph is determined from the radius of the figure produced.

To obtain the voltage, use is made of the calibration curve of the klydonograph. These curves for the range 3000 volts to 18,000 are shown in Plate III which shows the

PLATE III



relation between the radius of the figure obtained from the surge to the voltage which caused it.

Since each figure shows radii of different lengths the average of two or three of the longest is taken.

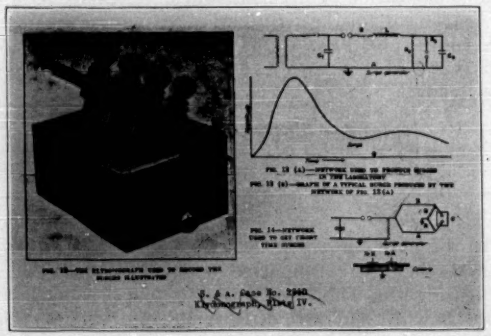
If the voltage is sufficiently high a spreading of the rays takes place. This begins at about 18,000 volts and a flash-over will occur if the voltage rises to 28,000 volts.

Figure *12* of Plate IV shows a laboratory form of klydonograph and Figure *13A* shows a form of current network used

in connection with the secondary of a transformer for alternating surges. Figure *13B* shows the graph of a surge produced by this network.

In order to study short time surges a network of the kind shown in Figure *14* is used. This consists of a condenser which is discharged through a sphere gap into a resistance from which two aërial lines *A* and *X* are carried. These

PLATE IV



lines are of variable lengths and their ends are connected to circular klydonograph terminals 3/8 inch in diameter, which are placed in contact with the film of the photographic plate.

The first simple form of klydonograph has been developed into a form that gives a continuous graphic record containing detailed information in regard to the magnitude of the surges, the time when they took place, the polarity, steepness of wave front, direction of travel and whether the surge was oscillatory or not.

To secure these results it is necessary that the improved instrument shall be provided with a clock mechanism by which a roll of sensitized film can be moved at a known rate and upon which the figures caused by the surges can be impressed for future study.

Very satisfactory opinions of the practical operation of this instrument have been received from users.

The opinion is also expressed that before any general rules can be formulated from the results obtained, the records of several instruments operating for a number of years will be needed.

The Institute is of the opinion that the klydonograph is an instrument that can be used to advantage in the study and interpretation of the surge phenomena of transmission lines.

In consideration of his successful development of a practical instrument for the recording of voltage surges on transmission lines THE FRANKLIN INSTITUTE awards its EDWARD LONGSTRETH MEDAL to MR. JOHN F. PETERS, of Swissvale, Pennsylvania.

THURLOW BACKWATER SUPPRESSOR.

HALL OF THE INSTITUTE,
PHILADELPHIA, May 4, 1927.

No. 2858.

The Franklin Institute of the State of Pennsylvania, acting through its Committee on Science and the Arts, investigating the Thurlow Backwater Suppressor, invented by Mr. Oscar G. Thurlow, of Birmingham, Alabama, reports as follows:

This device was developed and patented by Oscar G. Thurlow, of Birmingham, Alabama, for use in connection with hydraulic power plants and has for its objects:

(1) To increase the capacity of the turbines under conditions of high backwater in the tail race;

(2) To reduce the cost of installation by adapting the power house to form a part of the spillway.

United States Patent No. 1,594,578 was granted to Mr. Thurlow on August 3, 1926.

In many hydroelectric plants there is considerable loss in power output during high flow conditions of the river when water is being wasted over the spillway. This loss in power is due to the reduction in the effective head on the turbines, as under flood conditions, the tail water, or level below the dam, rises much more rapidly than the head water above the dam, resulting in a reduction of head on the turbine. The

output of a hydraulic turbine varies as the three-halves power of the ratio of the heads and hence an increased reduction in head results in a proportionately greater reduction in the horsepower output.

It is thus seen that in periods of excess flow when greater potential power is available in the river than under normal flow conditions, the actual output from the turbines is reduced below their normal output and the excess flow which cannot be passed through the units is wasted over the spillway.

In certain cases such as isolated plants serving districts which have no other source of power supply, such a reduction in output is a serious matter and may materially affect the value of the plant, as the primary power available all year demands the highest price and secondary power available only at times can only be sold at a reduced rate.

Another important consideration in the construction of a hydraulic plant is the provision of sufficient spillway section to take care of the flood conditions. Many factors are involved under this heading which vary in different localities, such as the amount of flood, length of dam, contour of surrounding country, etc. In many cases, it is desirable to use the entire length of the dam as spillway. If this is done, the powerhouse must be located independent of the dam, such as the construction adopted at the Holtwood dam on the Susquehanna River. It is usually desirable and economical however to build the powerhouse as part of the dam and this construction reduces the available spillway section.

In Mr. Thurlow's invention, he builds the powerhouse as part of the dam, but so arranges the turbine pits that in times of flood a portion of the water passes over or around the turbine pits and thence over the spillway, thus taking advantage of the entire length of the dam for spillway.

Water flowing over a dam having a downstream face of ogee section leaves the apron in a thin sheet at high velocity. At a point below the dam the water rises turbulently forming the so-called "standing wave" or "hydraulic jump" which is a well-known phenomenon. Formerly the energy in this overflow sheet has been regarded as solely of a destructive nature, but Mr. Thurlow in his device makes use of the energy of the spillway water by directing it over the turbine

draft tube orifices, thus removing the accumulation of backwater at this point, and increasing the effective head on the turbines.

A formula has been developed by Mr. Thurlow showing the relation between the total water column balanced in the tail-race (h) and the depth of water above the spillway (d) and the total head (H), so that $h = \sqrt{.83 d^{3/2} H^{1/2}}$. This formula was developed by equating the static pressure which the jet of water flowing over the spillway is capable of supporting to the static pressure exerted by the column of backwater h' high, and inserting in this equation for the quantity of water flowing over the spillway, the value of this flow expressed in depth of water over the spillway (d) using the Francis weir formula. In deriving this formula, all losses due to friction of moving water over the spillway are neglected, but as these losses are small, this formula gives approximately the maximum height of backwater (h), which a given depth of water over the spillway (d), falling a given distance (H), is capable of removing from over the draft tube orifices.

It was found from experiments that the river channel below the standing wave must be finished with neat concrete for a distance of five to six times the height of the standing wave for heads varying from 30 to 100 feet. This is necessary for good performance and in order to form as perfect a standing wave as possible. It was found by the experiments that if the river channels were not properly protected by concrete for the necessary distance beyond the standing wave, heavy under-currents were likely to occur which might very easily scour the river bed badly or undermine the protecting apron.

The provision of this concrete covering on the river bed below the standing wave is of particular importance if the rock strata are poor, as any open seams will allow the high static pressure beyond the wave to be transmitted back under the protecting apron causing an uplift. In fact, this uplift must be given attention even when the apron is carried out to the desired point and provision made for securely anchoring the apron.

APPLICATION OF SUPPRESSOR TO MITCHELL DAM PLANT.

Due to the splendid results obtained on the model tests, the Alabama Power Company decided to apply the backwater suppressor to the hydroelectric installation at Mitchell Dam on the Coosa River, Alabama, located about sixty miles southeast of Birmingham.

The plant was built during 1922, the initial installation consisting of three vertical shaft hydraulic turbines, each rated at 24,000 horsepower under 70 ft. head, at 136 r.p.m., direct connected to 18,000 K.W. electric generators.

In order to permit the installation of the backwater suppressor each unit was built on a separate foundation in the river on the upstream side of the dam and the draft tube discharge was carried through the dam directly under the spillway section. This arrangement also made available the entire length of the dam for spillway in times of high water, and as the river is comparatively narrow at this point, this was an important consideration.

In addition to the backwater suppressor, a number of novel features were incorporated in the design. For example, the usual powerhouse building is entirely eliminated and the generators covered by a low roof designed in two sections, mounted on rollers. In normal operation the generator room is protected from the weather, but in case of necessity this roof can be opened and a crane used to handle the large parts of machinery.

A single gantry crane travels over the entire length of the powerhouse section and thus can handle the penstock gates, trash racks, stop logs, generators, turbine parts, transformers and electrical switches.

An outdoor transformer and high voltage switching station is located on the bridge supported over the spillway.

The water enters the turbines through a concrete penstock and semi-volute casing, discharging down a vertical symmetrical draft tube and thence passing through a horizontal discharge tunnel under the spillway.

The excess water passes around the concrete structures containing the units and thence over the spillway, discharging immediately over the draft tube orifices, suppressing the backwater level at this point. The amount of water passing

over the spillway is controlled by three Tainter gates at each unit. These gates are raised by motor driven operating gearing located just below the bridge. In all, 26, of these spillway gates are furnished, each 15 ft. high and 30 ft. wide. These gates are capable of passing the maximum flood water known to have existed in the past.

In consideration of the novelty and ingenuity of this design, the careful investigation of the various elements affecting such design, its usefulness in increasing the potential output of hydroelectric plants under certain conditions and its successful application, THE FRANKLIN INSTITUTE awards its HOWARD N. POTTS MEDAL to MR. OSCAR G. THURLOW, of Birmingham, Alabama, for his Backwater Suppressor.

NOTES FROM THE U. S. BUREAU OF STANDARDS.*

RESEARCH ON PROPERTIES OF STEAM.

The development of the steam turbine during the present century has so greatly extended the range of temperatures and pressures at which steam is used, that the older data on properties of steam, which previously had been sufficient for technical requirements, were no longer adequate. To meet the situation, a conference of engineers and physicists was called in Cambridge, Mass., June 23, 1921, under the auspices of the American Society of Mechanical Engineers. This conference recommended a research program designed to produce more accurate data in the fields already covered, and to extend the investigations to higher temperatures and pressures. Harvard University, the Massachusetts Institute of Technology, and the Bureau of Standards were each to carry through portions of the program, which was to be supported by the American Society of Mechanical Engineers.

The Bureau of Standards, as its part of the program has been conducting an investigation of the properties of water and steam at the saturation limit. A thorough analysis of the problem was first made, which showed that a series of calorimetric measurements would yield coherent and consistent values for an important group of thermal properties of water and steam at saturation, and that by suitable design, the same calorimetric equipment could be used to determine certain properties of superheated steam.

On the basis of this analysis, the calorimetric equipment and the accessories were designed and constructed. The calorimeter consists of a miniature steam boiler, electrically heated, with provision for effective circulation of the water contained within it by means of a centrifugal pump. This boiler, which will hold about a pound of water, is carefully isolated so that heat interchange with its surroundings will be very slow, if there is a difference of temperature, and in operation, the envelop of the calorimeter is kept at the same temperature as the calorimeter by means of an oil bath.

* Communicated by the Director.

Thus the amount of energy added in an experiment can be accurately determined, as well as its effect in raising the temperature of water, or in evaporating the water.

Since hot water attacks many metals, the calorimeter and its parts are either made of, or protected with, metals or alloys which resist attack. The calorimeter itself is silver plated inside, and the parts inside are either pure silver, or of alloys of silver with gold, palladium, etc. These alloys were selected because they had the desired mechanical properties combined with immunity from attack by hot water.

Accessories include connections by which water can be transferred to or removed from the calorimeter, receivers in which water can be weighed, vacuum pumps for removing air from the calorimeter or from the space surrounding it, an oil bath with automatic temperature control at any desired temperature, a potentiometer and auxiliaries for measuring electric power input, connections to a standard clock for measuring time intervals, and platinum resistance thermometers and auxiliaries for temperature measurements. There is also a connection through a pressure capsule so that the pressure inside the calorimeter can be measured with a dead-weight piston gauge.

The calorimeter is designed for experiments up to 300° C. where the pressure is about 85 atmospheres or 1,250 lbs./in^2.

Measurements completed to date yield data on thermal properties of water and steam in the range 0° to 270° C. These results, combined with a few supplementary data from the work of other experimenters, are sufficient for a complete formulation of thermal properties of water and steam at the saturation limit, within the range given, and are offered as an American contribution which may serve as a foundation for a more complete steam table.

Research on properties of steam has also been actively pursued, for many years in Germany and England and more recently in Czechoslovakia also. During the summer of 1929, an international conference of research workers and others actively interested in the determination of the properties of steam, was held in London. This conference led to important results in promoting mutual understanding and it also suc-

ceeded in arriving at a basis for unifying the results obtained in the various countries. It abandoned the calorie and Btu as fundamental heat units, substituting the International kilowatt hour. It also agreed upon a skeleton table of the properties of steam which gives values with tolerances for the important properties, over the range useful in practice. It also stipulated that any steam table which was in agreement within the tolerances, with the skeleton table, and was thermodynamically consistent, should be regarded for the present as an international steam table.

A second conference is to be held in Berlin in June of this year. At this conference a wealth of new data including a considerable part of the results obtained at the Bureau of Standards, will be available for use in revising or completing the skeleton steam table.

A full account of the work done at the Bureau is to be published during the present year, in the Journal of Research.

Progress reports, giving the results of all the experimental work done in this country, have been published for a number of years, in Mechanical Engineering, in the February number. The skeleton steam table and the report of the London conference, appeared in Mechanical Engineering for February, 1930. The American Society of Mechanical Engineers has also published a Mollier Chart for steam and a provisional steam table. These publications can be purchased from the office of the Society, 29 West 39th Street, New York, N. Y.

AIRCRAFT ENGINE IGNITION SHIELDING.

Airplane engine ignition shielding is necessary in order to permit full and efficient use of the sensitive aircraft receiving sets now available. Ignition shielding consists of so confining the electrical fields of the ignition system that no interfering signal due to ignition can be induced in the radio receiving circuits. The work of the bureau toward the development of a practical ignition shielding assembly is described in the March number of the Bureau of Standards Journal of Research.

The entire electrical system of the ignition must be encased in a high conductivity metallic shield. This requires that the

magnetos be provided with such metal covers as will completely enclose the distributing heads. The booster magneto outlet must also be covered. All distributing wires must be enclosed in a metal casing, the spark plugs must be shielded, and the booster leads and leads to the ignition switch, including the ignition switch itself, similarly treated.

Enclosing the high voltage electrical system of the ignition in a closely surrounding grounded metallic casing introduces certain problems of mechanical and electrical design. It is much more difficult to secure a shielding assembly which will not adversely affect the engine ignition system than to obtain complete freedom from interference for the radio equipment.

The first shielding installation tried made use of the original ignition manifold supplied with the engine. The distributing leads were individually shielded with closely woven copper braid and threaded through the manifold. Shielded spark plugs were employed, the shield being incorporated as an integral part of the spark plug. The magneto distributor blocks were individually covered with metal cans accurately fitted to the magneto to within 0.005 inch. The ignition switch and the terminal block of the booster magneto were enclosed by metal covers. Shielding the switch and booster leads by means of closely woven copper braid completed the installation.

From a radio point of view the installation described was satisfactory. Not the slightest trace of ignition interference could be heard in the head telephones when using a receiving set having an overall voltage amplification of about 3,000,000 to 5,000,000. Practical difficulty was, however, experienced. The magneto and spark plugs were not sufficiently protected from water. Short-circuiting of the magneto distributors and of the spark plugs during rain occurred. Moreover, breaking of the small wires of the shielding braid, either in manufacture or in use, served to destroy the lacquered finish on the surface of the high tension cable and produced points of brush discharge owing to the interior current of the cable. The abrasive action of the metal on the cable also tended to injure the lacquered finish. The finish is necessary to protect the rubber of the cable against corona ozone and against the effects of oil and moisture. When the lacquer finish was

destroyed deterioration of the rubber occurred, resulting in faulty ignition.

In the next installation some of the difficulties outlined above were corrected. The spark plugs were made water-tight by means of a special elbow fitting incorporated in the plugs. A complete waterproof cover for each magneto was employed, with a single outlet for the nine ignition distributing leads. A complete ignition harness was used with the distributing leads grouped together in a single ring. The nine leads to each magneto were also grouped. A closely woven copper shielding braid surrounded each group of wires, liquid-proof loom being used as a cushioning fabric between the groups of wires and the copper braid. Braiding the individual leads from the ignition harness to the spark plugs was still employed.

This shielding assembly proved somewhat more durable in service than the first type described. The grouping of the leads and use of shielding braid around the groups rather than on individual leads considerably relieved the added dielectric stress on the ignition cable because of shielding. The portions of the ignition cable from the shielding harness to the spark plugs being directly covered with copper braid continued to be a source of trouble. Two installations were made, one lasting 30 hours and the other 50 hours before faulty ignition occurred.

The need of some method of shielding the individual leads from the ignition manifold to the spark plugs other than by the direct application of metal braid over the high tension cable was demonstrated. The third shielding assembly tried accomplished this by the use of liquid-tight flexible metal tubing, with a surrounding layer of copper braid to insure efficient shielding. A metal manifold was employed with unshielded high tension cable drawn through it in the usual way. To complete the shielding of the ignition leads from the manifold to the spark plugs each lead was enclosed in a suitable length of flexible metal tubing (provided with water-tight fittings at each end to connect to the manifold and spark plug). The groups of leads from the manifold to the magneto outlet were similarly enclosed in flexible metal tubing of proper size. This larger tubing was connected to the manifold

by a water-tight fitting and to the shielded magneto by means of an elbow fitting. This elbow fitting also provided outlet for the booster and switch leads. The use of flexible metal tubing was extended to the shielding of these leads.

This shielding assembly has proved satisfactory in operation and gives promise of fulfilling all the requirements for a safe ignition shielding assembly. It is now available in commercial form for the Wright J-5 and J-6 and for the Pratt and Whitney Wasp engines.

IMPROVEMENT OF RADIO MEASUREMENT METHODS.

The Bureau of Standards is trying to improve the methods of radio measurement and is investigating applications of these measurement methods to technical problems. In this connection, measurements have been made recently of the radio-frequency properties of insulating materials and of the characteristics of radio receivers.

At the request of another government department, a study of radio properties of mica was started. This includes among other properties the measurement of the phase difference, which is a measure of power loss in the material. This power loss is extremely small in mica. These measurements require the determination of very small changes in current, and interference from external sources may completely mask the effect which is looked for. To eliminate such interference a room completely enclosed with special copper screening has been built inside one of the rooms of the radio building. This shielded room makes it possible to carry on these and other very delicate measurements without any electrical disturbance.

In order to assist other government departments in the preparation of specifications for the purchase of radio receivers, and in the testing of radio receivers of different types, the bureau has installed equipment for making sensitivity, selectivity, and fidelity measurements at all frequencies from 15 to 4,000 kc. By the use of this equipment, tests are made throughout this frequency range which are similar to the tests outlined for broadcast receiving sets by the Institute of Radio Engineers.

There have been no publications on these lines of work, and

none are planned as yet. When the publications are prepared and issued they will be announced in these columns.

APPLYING THE VISUAL DOUBLE MODULATION TYPE RADIO RANGE TO THE AIRWAYS.

A paper of the above title, by H. Diamond, was published in the February issue of the Bureau of Standards Journal of Research. This paper deals with the methods which can be used for changing the angles between the courses worked out by radio range beacons and thereby obtaining alignment of the courses with the actual airways converging upon the airport where the radio range beacon is installed. A mathematical analysis is made of each method employed, and the amount of angular variation possible is tabulated. Sample calculations are made for actual airway routes to demonstrate the several methods. A number of circuit arrangements are described by which a single visual type radio range beacon may serve two, three, or four courses radiating from a given airport at arbitrary angles with each other.

Further details are given in the paper, which is available separately as Bureau of Standards Research Paper No. 148, obtainable by purchase from the Superintendent of Documents, Government Printing Office, Washington, D. C.

PHYSICAL PROPERTIES OF ELECTRICALLY WELDED STEEL TUBING.

The bureau has recently completed an investigation of the physical properties of sheet steel tubing made by the Johnston electric-welding process. In forming the longitudinal seam, the metal along a narrow band at the seam is brought to a welding temperature by passing an electric current through it, and the edges are then forced together by mechanical fixtures.

As the tubing comes from the welding machine it has a small burr on the inside and outside at the weld, and its outside diameter is from 0.02 to 0.04 inch greater than the nominal size.

The tubes included in this investigation varied in outside diameter from $\frac{5}{8}$ inch to 3 inches and in wall thickness from 0.028 (22 B. W. Gage) to 0.120 inch (11 B. W. Gage), inclusive.

They were tested in tension, in compression, in torsion, hydrostatically, in tension of weld of circumferential strips, for hardness, and for flanging. A comparative microscopic examination of the structure of the metal was made at the weld and at other points around some typical tubes.

The significant tests of the weld were the torsion, hydrostatic, and tension of circumferential strips with the weld in the center as well as outside of the tested section. In the torsion test the buckling (failure) did not begin at the weld in any of the specimens. The positions of the fractures in the hydrostatic specimens and in the tensile specimens of circumferential strips show that the strength, except in the case of the swaged-annealed tubes, does not depend upon the strength of the weld but upon the strength of the metal not affected by the welding operation (base metal). The tensile strength of the weld of the swaged-annealed tubes is about 90 per cent. of the strength of the base metal.

This investigation will be fully described in the April number of the Bureau of Standards Journal of Research.

RAYON AS A PAPER-MAKING MATERIAL.

Laboratory tests were conducted at the bureau to determine the paper-making quality of rayon when subjected to the same treatment as rags in the production of fine papers. Cotton and linen fibers are commonly used for papers such as bonds and ledgers on account of their strength and durability. In addition to their value in these respects they are used in the manufacture of papeteries and other fine writing papers also to impart softness and other desirable characteristics of surface and finish. The presence in rag stock of any fiber which would adversely affect these qualities of the paper is undesirable. Since increasing amounts of rayon are being found in the rags and textile waste used in rag-paper manufacture the test data are believed to be of value to that industry.

Samples representative of the four general processes of rayon manufacture—viscose, nitrocellulose, cuprammonium, and cellulose acetate—were included in the tests. The equipment employed in the preparation of the pulp and its subsequent conversion into paper was on a laboratory scale

and is that used at the bureau for the preliminary tests of all paper-making materials under investigation. The caustic soda and lime process were each used in the cooking operation. Paper-making stock was prepared from the pulp, alone and in admixture with sulphite pulp, and converted into paper on a sheet mold.

Owing to loss of strength when wetted the rayon filaments tended to break into short lengths during the preparation of the paper-making stock without the fibrillation and fraying necessary for good felting properties. As a consequence the all-rayon sheets lacked the strength to withstand the handling necessary in the pressing and drying operations, and the pliability characteristic of rag papers. Likewise, sheets made of rayon in admixture with sulphite pulp were considerably weaker than those made from sulphite alone.

The test data indicate that rayon is valueless in the rag stock for high-grade papers and may actually be detrimental to their quality.

A complete report of this investigation, prepared by M. B. Shaw and G. W. Bicking, was published in the February, 1930, number of the bureau Journal of Research, as Research Paper No. 143. Separate reprints may be purchased from the Superintendent of Documents, Government Printing Office, Washington, D. C., at 5 cents each.

MINERALOGICAL CHANGES IN FIRE CLAYS.

Recent experimental work with fireclays has led to an interesting observation in which it was noted how certain reactions proceed when such clays are fired at comparatively high temperatures. It has been known for some time that mullite and free silica develop during the firing of clays in certain temperature ranges and that fluxes bear an important relation to that development. The bureau is now able to show how the linear thermal expansion is directly affected by the formation of mullite and the development of free silica accompanying it. The formation of mullite from clay may be expressed as follows:

$$3(Al_2O_3 \cdot 2SiO_2) + \text{heat} = 3Al_2O_3 \cdot 2SiO_2 + 4SiO_2.$$

Linear thermal expansion measurements from 20° to 900° C. were made of three different fireclays containing very little, if any, free quartz previous to firing and whose alumina content on a water-free basis was approximately 45 per cent. These measurements and petrographic examinations were made after firing the clays at the following temperatures: 1120, 1180, 1260, 1400, and 1525° C. The rate of expansion of the clays after firing at 1120° C. was practically uniform throughout the entire range studied, and the petrographic examinations failed to show the presence of mullite. As the temperature of firing was increased (1180 and 1260° C.), both the expansion and mineralogical composition showed decided changes. The linear thermal expansion observations which show no longer a uniform rate of expansion below 200° C. indicate that the silica set free as a result of the development of mullite appears to be in the form of cristobalite with possibly a small percentage of tridymite. The petrographic examinations show the presence of mullite and a glassy matrix too small in quantity to identify. After firing at 1400° C. the high rate of expansion below 200° C. has decreased considerably and after the firing at 1525° C. the rate throughout the entire range studied (20° to 900° C.) has again become uniform though somewhat lower than that obtained after the clay was fired at 1120° C. The petrographic examination shows that the clay after firing at 1525° C. is composed entirely of mullite and a glassy matrix material. The results of the thermal expansion measurements, together with the information obtained in the petrographic examination, would indicate that the cristobalite has been converted to glass.

INDEX OF REFRACTION VERSUS COMPOSITION OF SODA-LIME GLASSES.

In connection with the Bureau's work on optical glass, it seemed desirable to determine if possible numerical relations between the chemical composition of glasses and their refractive index which would cover a larger field than those previously investigated.

A series of "pure" glasses ranging in composition from about 50 per cent. each of silica and soda to 80 per cent. silica—

20 per cent. soda were made. Also, similar glasses were made in which from 3 to 15 per cent. lime was substituted for soda.

The raw materials from which these glasses were made were selected with the greatest care; the soda and lime were chemically pure and the silica was obtained from large colorless quartz crystals. The glasses were melted in a platinum crucible, heated in a platinum resistance furnace.

The refractive indices (N_d) of these glasses were measured by an immersion method which gives an accuracy of approximately 0.0003. All of the glasses were carefully analyzed, in some cases the percentage of soda being obtained "by difference."

A study of the data led to the conclusion that the index of refraction of soda-silica and soda-lime-silica glasses in the range between the ordinary commercial glasses and the exceedingly soft glasses can be computed from the following equations in which P is the percentage of the respective constituents:

$$N_d \text{ due to } SiO_2 = 0.042712(P)^{0.79487} - 0.2,$$

$$N_d \text{ due to } Na_2O = 0.0153(P) - 0.00837,$$

$$N_d \text{ due to } CaO = 0.017219(P) - 0.00103.$$

Using the above expressions, the index of refraction of 33 of the 35 experimental glasses can be computed to better than 0.001 and in the majority of cases to about 0.0005.

REVISED CIRCULAR ON ARCHITECTURAL ACOUSTICS.

Circular No. 380 (a revision of Circular No. 300 of the Bureau of Standards) on architectural acoustics was released early in March and contains valuable information on the design of auditoriums.

The circular points out that the usual acoustical defects of auditoriums are echo, dead spots, and reverberation. Reverberation (including echo, which is a particular kind of reverberation) is a serious defect since the prolongation of one sound such as a musical note or spoken syllable, may interfere with the next sound, producing hopeless confusion. This is likewise a difficult defect to remove, and should be prevented

by foresight in construction. Whenever possible, the architect should avoid circular halls and smooth, hard-finished walls. Surfaces, such as the ceiling and even the proscenium arch in theaters, should be broken up into irregular areas which will scatter the reflected sound.

Dead spots and sound foci result as a consequence of reverberation-producing conditions and may be treated in the same way. Balconies often require special treatment, and if low and deep are almost certain to be unsatisfactory.

A certain amount of reverberation, or prolongation of a sound, is desirable provided the reverberation does not last long enough to interfere with the next succeeding sound. A table of acceptable limits of reverberation time for rooms of different volumes, and formulas for calculating the reverberation time are given. Values are likewise given for the sound absorbing powers of different materials as compared with an open window which constitutes an almost perfect absorber.

Practical general directions for designing an auditorium are included, and it is pointed out that rooms of certain sizes are best for specific purposes. Thus a theater should be of moderate size, while an auditorium for musical numbers may be much larger. In case an orchestra is reinforced by an organ due allowance must be made or the organ may drown out the other instruments. Great care should be used in selecting the interior finish so as to reduce the reverberation time to a proper value. There should be no difficulty in selecting suitable trimmings, since sound absorbing materials of many different kinds are now available commercially.

Copies of this circular may be obtained from the Superintendent of Documents, Government Printing Office, Washington, D. C., at 5 cents each.

THE FRANKLIN INSTITUTE.

STATED MONTHLY MEETING, MARCH 19, 1930.

The regular monthly meeting of The Franklin Institute was called to order at eight-fifteen P.M. by Mr. Nathan Hayward, President, who called upon the Secretary for a report concerning the business of the meeting.

The Secretary announced that the minutes of the February meeting had been published in full in the March JOURNAL of the Institute, which is now in the hands of the members. He thereupon moved that the minutes be approved as printed in the JOURNAL. The motion was seconded and was adopted unanimously.

The Secretary then reported to the meeting that former Senator George Wharton Pepper had accepted the Chairmanship of the General Committee in charge of the campaign for the erection of the Benjamin Franklin Memorial and Franklin Institute Museum on the Parkway, that all the activities of the Committee had been greatly stimulated, that the organization for the conduct of the campaign was rapidly approaching completion, and that a highly successful outcome seemed assured. He stated that he had pledged the heartiest coöperation in the campaign from the membership of the Institute and called upon those members of the Institute who were present to make his pledge good. He stated that every friend and member of the Institute would have an opportunity later, by service as well as by gifts, to contribute to this great cause which means so much for Philadelphia.

The Secretary reported that since the last regular meeting four Resident members and two Non-Resident members had been elected to the Institute, but that, unfortunately, three members had died during the same interval. He then made an appeal to all members of the Institute who had friends or acquaintances interested in this old Society to submit their names for membership. He outlined the purposes of the Institute and the requirements for membership and again urged the members to attempt to increase the number in the Society. He offered to send application blanks to anyone whose name and address might be submitted to him.

The President announced there was no further business to be considered and introduced as the speaker of the evening Dr. Ralph H. McKee, Professor of Chemical Engineering in Columbia University, who spoke delightfully on "Research in the Cellulose Industries." Dr. McKee showed how it had been possible to make better cotton textile without additional cost, described the method of producing a wool-like product (artificial wool) from a low grade textile fiber, and finally described at length the results of efforts to produce by hybridization trees of unusually rapid growth. He showed the results which had been obtained by these efforts by which trees giving more than ten times as much pulp wood per acre as is now given could be developed.

The paper was followed by an animated discussion in which a considerable number participated.

The meeting closed at nine-forty, with an enthusiastic vote of thanks to Dr. McKee.

HOWARD MCCLENAHAN,
Secretary.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, March 12, 1930.)

HALL OF THE COMMITTEE,
PHILADELPHIA, March 12, 1930.

MR. HENRY B. ALLEN *in the Chair.*

The Chairman announced the election by the Board of Managers on February 19, 1930 to membership in the Committee of

MR. FRANK B. EVANS, JR., Assistant Vice President, The Bell Telephone Company of Pennsylvania.

The following reports were presented for final action:

No. 2909: Work of Mr. Irving E. Moultrop in the Application of High Pressure Steam Boilers.

This report recommended the award of the Elliott Cresson Medal to Mr. Irving E. Moultrop, Designer and Chief Engineer of the Boston Edison Company, Boston, Massachusetts:

"In consideration of the engineering skill displayed in the design and construction of the high pressure installation at the Edgar Station, whereby a marked advance in the art of steam electric generation was accomplished and a reliable source of information for advance prediction, not only of performance but of initial cost of such stations was made available and in which courage of high order was manifested."

No. 2917: Work of Doctor Theodore Lyman,

This report recommended the award of the Elliott Cresson Medal to Doctor Theodore Lyman, Hollis Professor of Natural Philosophy, Emeritus, Harvard University,

"In consideration of his extension of our knowledge of spectral phenomena in a region of great experimental difficulty and of high theoretical interest."

The following reports were presented for first reading:

No. 2809: Method and Apparatus for Dry Cleaning Coal.

No. 2923: Clark Medal.

GEO. A. HOADLEY,
Secretary to Committee.

MEMBERSHIP NOTES.

ELECTIONS TO MEMBERSHIP.

(Stated Meeting, Board of Managers, March 19, 1930.)

RESIDENT.

MISS RUTH CUSTIS KITCHEN, Graduate Student, Bryn Mawr College. For mailing: The Cambridge, Germantown, Philadelphia, Pa.

MR. HENRY M. LYMAN, Power Engineer, 704 North Fortieth Street, Philadelphia, Pa.

COLONEL ROYAL MATTICE, Metallurgical Engineer, 2235 Vine Street, Philadelphia, Pa.

MR. CLINTON E. SHAW, Instructor, Northeast High School, Philadelphia, Pa. For mailing: 2030 Spruce Street, Philadelphia, Pa.

NON-RESIDENT.

MR. WILLIAM C. PETERMAN, Electrical Engineer, Assistant Chief Engineer, International Communications Laboratories, 67 Broad Street, New York City.

MR. DAVID P. WHEATLAND, P. O. Box 305, Salem, Mass.

CHANGES OF ADDRESS.

MR. ISAAC W. BYERS, Iron River, Michigan.

MR. W. A. CONVERSE, 310 South Michigan Avenue, Chicago, Illinois.

MR. CHESTER L. DAVIS, 1418 Main Street, Keokuk, Iowa.

DR. HAROLD W. ELLEY, Room 8114 DuPont Building, Wilmington, Delaware.

DR. ROBERT GRIMSHAW, 128 West Seventy-eighth Street, New York City.

MR. WALTER G. HENSEL, c/o Thomas Cook and Son, Ltd., Unter den Linden 22, Berlin, W. 8, Germany.

MR. WILLIAM HOSKINS, Mariner and Hoskins, 2009 Harris Trust Building, 111 West Monroe Street, Chicago, Illinois.

DR. WALTER M. MITCHELL, 939 Fulton Road, N. W., Canton, Ohio.

MR. JOSEPH M. STEEL, 124 North Fifteenth Street, Philadelphia, Pa.

MR. RUSSELL G. THOMPSON, Electromatic Typewriters, Inc., 45 Crouch Street, Rochester, N. Y.

MR. WARREN P. VALENTINE, Hilltown Road, Line Lexington, Pa. Business address: 713 Atlantic Building, Philadelphia, Pa.

DR. ROBERT J. ANDERSON, 2195 Lincoln Avenue, N. W., Lakewood, Cleveland, Ohio.

NECROLOGY.

Mr. Jacob D. Cox, Cleveland, Ohio.
Mr. W. A. Rosenbaum, New York City.
Captain E. R. Wood, Jr., Philadelphia, Pa.

LIBRARY NOTES.

RECENT ADDITIONS.

ALEXANDER, JEROME. Colloid Chemistry; Principles and Applications. Third edition. 1929.

American Institute of Steel Construction. Steel Construction. First edition. 1930.

American Pharmaceutical Association. Year Book. Volumes 11–14, 1922–1925. Four volumes. 1924–1927.

BELL, H. S. American Petroleum Refining. Second edition. 1930.

BENNETT, J. M. Roadside Development. 1929.

BLUM, WILLIAM, AND GEORGE B. HOGABOOM. Principles of Electroplating and Electroforming (Electrotyping). Second edition. 1930.

Brassey's Naval and Shipping Annual. 1930.

British Journal Photographic Almanac and Photographer's Daily Companion. 1930.

BYWATERS, H. W. Modern Methods of Cocoa and Chocolate Manufacture. 1930.

CALVERT, ROBERT. Diatomaceous Earth. 1930.

COFFIGNIER, CH. Colours and Varnishes. Translated from the French by A. Harvey. 1925.

DALBY, W. E. The Balancing of Engines. Fourth edition. 1929.

DEVRIES, LOUIS. German for Chemists. 1929.

FLEURY, PAUL. The Preparation and Uses of White Zinc Paints. Translated from the French by Donald Grant. 1912.

Forschungsinstitut der cechoslovakischen Zuckerindustrie in Prag. Bericht. Bd. 30–32. Three volumes. 1927–1929.

Foundrymen's Handbook. Second edition. 1929.

FOX, CYRIL S. Bauxite. 1927.

FOX, J. J., AND T. H. BOWLES. The Analysis of Pigments, Paints and Varnishes. 1927.

FREEMAN, BENJAMIN, AND FREDERICK G. HOPPE. Electroplating with Chromium, Copper, and Nickel. 1930.

GLAUERT, H. The Elements of Aerofoil and Airscrew Theory. 1926.

GOULD, RUPERT T. Oddities: a Book of Unexplained Facts. 1928.

HAYWARD, CARLE R. An Outline of Metallurgical Practice. 1929.

HILDITCH, T. P. Catalytic Processes in Applied Chemistry. 1929.

HODGMAN, CHARLES D., AND NORBERT A. LANGE. Handbook of Chemistry and Physics. Fourteenth edition. 1929.

ISEMAN, JOHN W., Editor-in-Chief. The Aviation Manual. 1930.

JENNISON, FRANCIS H. The Manufacture of Lake Pigments from Artificial Colours. Second revised edition. 1920.

JONES, M. W. The Testing and Valuation of Raw Materials Used in Paint and Colour Manufacture. 1900.

LATIMER, WENDELL M., AND JOEL H. HILDEBRAND. Reference Book of Inorganic Chemistry. 1929.

METCALF, LEONARD, AND HARRISON P. EDDY. Sewerage and Sewage Disposal. Second edition, revised by Harrison P. Eddy. 1930.

MILLER, HENRY W. The Paris Gun; the Bombardment of Paris by the German Long Range Guns and the Great German Offensives of 1918. 1930.

MITKE, CHARLES. Mining Methods. First edition. 1930.

NAETHER, CARL A., AND GEORGE FRANCIS RICHARDSON. A Course in English for Engineers. Volume I. 1930.

OSBORN, FREDERICK A. Physics of the Home; a Textbook for Students of Home Economics. Second edition. 1929.

Oxford University, Institute for Research in Agricultural Engineering. A Report on the Development and Costs of the Oxford Process for the Production of Sugar from Sugar Beet. 1929.

PARRY, ERNEST J., AND JOHN H. COSTE. The Chemistry of Pigments. 1902.

PETIT, G. The Manufacture and Comparative Merits of White Lead and Zinc White Paints. Translated from the French by Donald Grant. 1907.

REID, E. EMMET. College Organic Chemistry. 1929.

RIGGLEMAN, JOHN R. Graphic Methods for Presenting Business Statistics. 1926.

ROBISON, S. S. Robison's Manual of Radio Telegraphy and Telephony for the Use of Naval Radiomen. Revised by Capt. S. C. Hooper. Eighth revised edition. 1928.

S., M. E., AND K. R. S. Sir Joseph Wilson Swan, F. R. S.; a Memoir. 1929.

SABIN, ALVAH HORTON. House Painting, Glazing, Paper Hanging, and Whitewashing. Fourth edition, enlarged. 1929.

SEARLE, ALFRED B. An Encyclopædia of the Ceramic Industries. Volume II. 1930.

STARR, EDGAR M., AND EDWIN G. OLDS. Vocational Mathematics. 1930.

SULLIVAN, J. W. N. The Bases of Modern Science. 1929.

SWETT, GEORGE W. Construction of Alignment Charts. 1928.

Taylor Society, H. S. PERSON, Editor. Scientific Management in American Industry. 1929.

U. S. Department of Commerce. Commerce Year Book 1926 and 1928. Four volumes. 1927–1928.

WILSON, P., AND G. W. WEBB. Modern Gramophones and Electrical Reproducers. 1929.

WOLFANGER, LOUIS A. The Major Soil Divisions of the United States: a Pedologic-Geographic Survey. 1930.

BOOK REVIEWS.

VOCATIONAL MATHEMATICS. By Edgar M. Starr and Edwin G. Olds, Assistant Professors of Mathematics in the Carnegie Institute of Technology. x–184 pages, illustrations, 12mo., cloth. Philadelphia, P. Blakiston's Son and Company, Inc., 1930. Price, $1.25.

Students in vocational schools generally have been expected to have such educational training as is given in our lower grade public schools. That is only a reasonable requirement for the prosecution of any sort of special study, yet it is a common experience that the majority of such students are decidedly deficient in

that prerequisite. In one way or another all arts and crafts deal with quantities, and it is obvious that proficiency in arithmetical operations is especially essential to their mastery. In harmony with these conditions the authors have prepared the present text to be included in courses of vocational study.

The book is in two parts, the first, Arithmetic, the second, Vocational Mathematics. The first part is a logical explanation of the principles of arithmetic with numerous illustrations and exercises. The second part deals with the mensuration of plane and solid geometrical figures and irregular figures including the properties of Pappus. There are final chapters on logarithms, plane trigonometry of the right angle and graphic representation. This section is likewise copiously illustrated with explanatory examples and diagrams, and provided with numerous practice exercises. The arithmetical complement, it is noticed, is used in the logarithmic work. Many computers now write negative characteristics as they are, their addition requiring no appreciably greater effort than the time-honored arithmetic complement. Vocational students who are always in a hurry to get into action, will not be discouraged by the dimensions of this book, yet will derive from its conscientious perusal the training in computation indispensable to successful study.

L. E. P.

A COURSE IN ENGLISH FOR ENGINEERS. Volume I, The Engineer's Speaking and Writing—in General. By Carl A. Naether, M.A., Associate Professor of English, University of Southern California, in charge of instruction in business writing. In collaboration with George Francis Richardson, Ph.D., with an introduction by J. A. L. Waddell, D.E., LL.D., 330 pages, illustrations, 8vo. Boston, Ginn and Company, 1930. Price, $2.60.

It is a gratifying sign of the times that within a short period two books have appeared directed to instructing students in professional schools in the correct use of English. One of these was reviewed in this JOURNAL a few months ago, and now another effort is before us, a more pretentious effort, for the octavo volume of over three hundred pages is to be followed by a second.

The problem of developing a careful attention to grammatical and syntactical forms in English is a very difficult one. English-speaking countries have no dominant authority to which disputed questions of spelling, and phrase may be referred. The great nations of continental Europe are satisfactorily controlled by organizations of scholars, which if not really official, are sufficiently respected by governments to have their rulings approved in the vast majority of cases. A striking instance of this unified control was the dropping of the silent "h" in the combination "th" in German words. This, directed by scholarly authority, has been universally followed by German writers. Authorities of equally high scholarship in Great Britain and the United States have advocated a few changes in the direction of simplified spelling of English words, but have obtained almost nothing except ridicule. One of the features of the dispute over the movement for spelling reform, which would be amusing if it were not disgusting, is the cry that the changes would take from our literature the spellings of Shakespeare and Milton.

The work in hand is much more than a dissertation on pronunciation and grammatical forms. It aims to equip engineers with ability to prepare their

scientific papers in literary form. It is in fact a comprehensive treatise on rhetoric. Many illustrations of methods of composition are given, the resources of English literature are drawn on extensively, these selections ranging from grave to gay, from lively to severe. In making such selections the personal tastes of the author will, of course determine, and others may find reason to disapprove. No extensive disapproval is to be here expressed, but the story on page 148 is entirely lacking in humor and the poem that follows it is without merit.

How far a book of this type will improve the quality of American professional literature is yet to be determined. The reviewer does not wish to be a bird of evil omen, but a very large experience in teaching in professional schools, makes him despair of securing much improvement. In the pages of one of our leading scientific journals, an experienced professor openly defended the use of the form "data is." What hope is there that in the institution at which that man teaches there will be any encouragement for attention to exact forms? It may be that the engineering profession, as yet, is more homogeneous as to racial types and preliminary training than the professions of medicine, dentistry, law and pharmacy, into which representatives of the races of southern and eastern Europe are now entering in such large numbers. If so, engineering schools will show a greater appreciation of English language and literature, thus giving a better medium upon which to cultivate the material of this book. After all, the work must begin in the home and in the elementary schools. It is especially in the latter that the rules of grammar and the principles of orthoepy and syntax should be taught. Great laxity prevails in this respect in American schools. Pupils are permitted to use many incorrect forms, to slur letters, to talk and write slang and to learn much text by rote without comprehension of meaning. Arithmetic is also very imperfectly inculcated.

Efforts at reform in these matters meet not only difficulties from the lack of systematic central authority to establish standards, but there is in our own country a spirit of insubordination to such control. An assertion of individuality is a dominant motive in the land. Is it far-fetched to ascribe this to the prohibition dispute? The suggestion may excite laughter or even condemnation, but it really seems as if the struggle over the great restriction experiment has aroused an exceptionally high claim of individual rights. The emotion has broken its original bounds and led to claims of independence in other fields of human action. The resistance to parking regulations is a phase of the same spirit.

The book is to be commended as an earnest effort to improve what needs much improvement and it is to be hoped that it may find satisfactory use in the fields for which it is intended, but any considerable reform in diction in American scientific papers is, it is to be feared, as far off as naval parity.

HENRY LEFFMANN.

PHYSICS OF THE HOME. A Textbook for Students of Home Economics. By Frederick A. Osborn, Ph.D., Professor of Physics, University of Washington. Second Edition, xiv–397 pages, 221 figures, 8vo. McGraw-Hill Book Company, Inc., New York, 1929. Price $3.00.

As the author aptly puts it: "'Physics of the Home' is offered as the first formal attempt to meet the needs of women students who want to know physics not for its own sake but for its real help in their daily life." The book contains information which should be of real interest and aid to the housewife. Many of

the facts and findings will attract more attention from her than from the student who, when taking the course, so often admits a mere academic interest.

No doubt a woman would be interested in knowing that the friction between the shoe and a cement floor is three times that for a waxed floor which might or might not be a bit too "slick" for dancing safely; that by taking long steps she works harder than if the steps were shorter; that when she gives her weight as 120 pounds, she really weighs 120 pounds 3 ounces, that is, upon reducing—to vacuo; that a vacuum cleaner will prevent dirt from penetrating into the rug but scarcely remove that already deeply imbedded; that when making candy, the syrup should be kept at definite temperatures depending on whether fondant, fudge, taffy or brittle is desired; that a pie crust dough mixed cold will be lighter than one made at a higher temperature; that if a young woman weighing 120 pounds confines herself to eating no more than ¼ pound of candy, and assuming 20 per cent. efficiency of conversion she has acquired sufficient energy to walk a distance of two miles; that it takes over six pounds of ice to thoroughly chill twenty quarts of punch.

The foregoing facts are just a few of the many interesting subjects dealt with in this book. A chapter on "Distribution of Heat" discusses the value of various types of insulation and methods for computing heat losses in a residence. The advantages and disadvantages of various types of heating systems are outlined and attention called to the problems of humidity regulation and ventilation. Considerable space is given to the subject of illumination in the home in which are described the lighting coefficients of various types of reflectors and the elimination of factors producing glare. Color is treated from both the physical and psychological standpoint for the purpose of supplying valuable information on color contrast and harmony in the home. Special attention is paid to the problem of heating with electricity. Efficiencies when heating a house, heating water, or cooking have been studied and interesting data furnished comparing the costs of heating by coal, gas or electricity. The book also contains many review questions and representative problems which go far toward giving the student or reader a firmer grasp upon the subject.

T. K. CLEVELAND.

THE BALANCING OF OIL ENGINES. By W. Ker Wilson, B.Sc. (London), Wh. Ex., Hons. Dip. Mech. Eng. Illustrated, xii-279 pages, 8vo, cloth. London, Charles Griffin and Company, Limited; Philadelphia, J. B. Lippincott Company, 1929. Price, $10.00 net.

The high speeds of modern machinery of all kinds have brought into prominence the necessity of a careful examination of a variety of problems in vibration engineering, and have stimulated in recent years an active investigation of the subject in its many aspects, and in particular to refinements in the balancing of reciprocating engines. In the analytical processes of determining the magnitude of unbalance and applying the possible correctives in the design of an engine, no methods of procedure of striking novelty have been recorded in the several works on the subject now extant since the notable contributions of Yarrow, Schlick and other contemporary investigators during the final years of the past and the early years of the present centuries. Balancing by experimental methods, however, has received of late years a considerable impetus, and final accurate balance,

particularly in automotive work, by the use of special machines is now extensively practiced.

The present work consists, essentially, of a more or less codified presentation of well established analytical principles applied to the practice of balancing of internal combustion engines. The fundamental process employed in the analysis is the utilization of the formulas for the horizontal and vertical components of the forces due to rotating masses and those of the inertia of the reciprocating masses in terms of the crank-angle. These formulas are stated without derivation. As to the absence of derivation of formulas, there is little doubt that many readers would welcome the inclusion of the steps in the derivation of the latter which, though not difficult, entails a bit of mathematical ingenuity. The work deals with engines of the Diesel vertical marine type with one up to eight cylinders. The first three chapters contain what may be classed as a preliminary survey of general principles and their application to the calculation of unbalanced forces and couples, vertically and horizontally, with a consideration of inertia forces and couples of higher order. These higher-order forces which appear as functions of multiple angles in the fundamental formulas are shown to be of importance at high speeds. Further along a striking illustration, with line-cut and half-tone of arrangement, is given of the balancing methods employed in overcoming objectionable secondary unbalance in an oil-engined passenger vessel. Numerous tables, with the crank-angle as the argument at fifteen-degree intervals, of the variables of the inertia forces and couples relieve the designer of much of the burden of computation.

Subsequent chapters contain complete analyses of single and multi-cylindered engines under the caption of cylinder-number. Among these some matter, which might more properly be placed in the early chapters, is included, as the division of the mass of the connecting-rod into reciprocating and revolving masses. The accurate secondary balancing of single-cylinder engines by the use of geared rotating masses and by the application of bob-weights or their equivalent is described and formulated. With the multi-cylinder engines formulas, are given formulas for the application of balance weights for the cranks in planes remote from the cranks. The forces acting at these points in the diagrams which accompany the text are lettered as the weights applied at these points and are indicated by vector force-lines. It would be helpful to write at those points also the forces which those arrowed lines represent. The mutual balancing effects and the influence of auxiliaries in multi-cylinder engines are very fully analyzed, and careful attention is directed to the stresses induced upon the shaft and frame by individually unbalanced but collectively balanced forces. This is illustrated by diagrams of these bending moments. A chapter on graphical methods applied to balancing problems, a consideration of matters of design in relation to balance, and the design of anti-vibration foundations are included in final chapters.

Despite the many tables, the numerous insets and the profusion of formulated matter the price of the book is unusually high. There is no bibliography, and since there is a considerable literature of the subject, the addition of references would be of value. In general the subject is presented in a form that will appeal to the practical designer and prove an efficient labor-saving document.

LUCIEN E. PICOLET.

INORGANIC QUALITATIVE CHEMICAL ANALYSIS. By Allan R. Day, Ph.D., University of Pennsylvania. 8vo, 197 pages. The Chemical Publishing Company, Easton, Pa., 1930. Price, $2.25.

There are many books of this type available, but they differ considerably in the stress laid upon the theoretical as compared with the practical phases. Before the general principles of physical chemistry were being taught, manuals of qualitative analysis were mostly descriptions for precipitation and filtering, with a few special tests for verification. As carried out in most chemistry courses, the work is sloppy, calculated rather to make the students careless than a guide to the attention to cleanliness and detail that is necessary in chemistry.

The reviewer has always felt that the common method was ill-advised and it is gratifying to learn from a note in the preface that there is now a tendency to decrease the time devoted to inorganic qualitative analysis in college courses. Nevertheless, the enormous increase in chemical data, the additions to methods of qualitative research, together with the increase in the list of elements, many formerly museum curiosities, now familiar in commerce, have extended the field and made the selection of methods more difficult.

The present work gives a large attention to the theory, explaining many of the procedures upon the basis of modern physical chemistry. Undoubtedly, a student using the work under the guidance of a competent instructor will master many important features of elementary analysis, but it is doubtful if any considerable proportion of students will enter from high schools with the mathematical knowledge necessary to appreciate the statements. However, this is not the author's concern. He has set forth in very satisfactory manner the principles of a standard course in inorganic qualitative analysis and the student will profit by it. On the practical side some exception may be taken. It is strange that in describing Marsh's test, no warning is given as to the danger from arsine. Reinsch's test is very imperfectly described. Applied carefully, it is one of the most convenient and satisfactory tests for arsenous compounds; arsenates are more difficult of detection by it than the book indicates. In connection with the too brief notice of the test, no mention is made that mercury, antimony and bismuth compounds may stain copper, nor is the important (and to the student, interesting) fact mentioned that the copper slip after coating with the arsenical deposit will yield on gentle heating a characteristic sublimate of octahedral crystals of arsenous oxide.

H. L.

ASBESTOS, A LIST OF REFERENCES TO MATERIAL IN THE NEW YORK PUBLIC LIBRARY. Complied by William B. Gamble, Chief, Science and Technology Division. 72 pages, 8vo, paper, New York, The New York Public Library, 1929. Price 50 cents.

That remarkable fibrous mineral, known as asbestos, which can be fashioned into fire-resistant woven fabric or mechanically resistant non-conductive material for electrical apparatus is of familiar and extensive application in the industries. Much of the literature which has grown about its rapidly acquired industrial importance is widely distributed in the files of technical societies or other like sources which are not readily found without laborious search. This bibliography which is

compiled from works relating to asbestos owned by the New York Public Library contains a comprehensive collection of papers on the subject in its varied aspects. In addition to those references, a list of United States, English, German and Canadian patents with their titles is given. A copious general index is also provided. The author has placed in his debt all investigators who may desire information on the subject with the least expenditure of time and labor, and provided a suggestive model of bibliographic service applicable to other fields of technology which may be commended to the thoughtful attention of directors of technical libraries.

LUCIEN E. PICOLET.

THE TERMINOLOGY OF PHYSICAL SCIENCE. By Duane Roller. 115 pages, 12mo, paper. Norman, Oklahoma. University of Oklahoma Press, 1929. Price $1.00.

In the specification of a quantity of any kind in physical science, it is of supreme importance that it shall invariably convey a meaning with unmistakable precision. The study of physical science has not always been the vast coöperative effort of world-wide extent that, in effect, it has become in modern times, and what confusion still persists in its terminology, incident to the isolated efforts to which it owes its early rise and even its later progress, is obviously enough inevitable. Modern investigators have done much to eliminate these uncertainties, but old habits of expression as well as of thought at an early period of study is an invaluable asset for future progress, and in preparing this critical review, the author has had in mind the needs of the teacher and the text-book writer and not those of the student for, as he points out, dialectic discussions have no place in the class-room.

Physical terms and their definitions, which is the major topic, occupies somewhat more than half the volume. Prefixes and suffixes, chemical elements, pronunciation, spelling and abbreviation are included in the remainder. The analysis is arranged systematically under the general classification of terms that should be discarded, and terms requiring careful definition or special comment. The discussion of those groups is followed by a glossary of terms which contains many convincing explanations and comments. Among the terms recommended for the discard are "actinic rays," "chemical rays," "centroid," "compound pendulum," "simple pendulum," "Marriotte's law," "moment of inertia," "permanent gas" and "uniform acceleration." Some of these will probably not go out of use without a struggle. Moment of inertia, for example, particularly regarded as a geometric quantity, is a most familiar term among all classes of structural designers and appears in many recent texts. That and, for a similar reason, "compound pendulum" and "simple pendulum" may be hard to dislodge. In books written in English, Marriotte appears already to have received the "coup de grace" and in some French books also. Among his compatriots there is naturally disappointment at his exclusion for the lesser but earlier work of Boyle, but such is the law of prior occupation. Common prefixes and suffixes and the names of the chemical elements are similarly tabulated with information on their meaning, origin and examples of their occurrence.

Coming to the pronunciation of words used in science, the task of adjustment is of a more subtile character, but in the few pages devoted to that topic many

valuable comments will be found, and the list of commonly mispronounced words which follows should be helpful even to many who are reasonably well trained in English syntax. "D'Alembert" given in the list is spelt phonetically "da lan ber." The last syllable might better have been written "bear."

The work is a most interesting and suggestive review of the difficulties encountered in the terminology of physical science.

L. E. P.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

Report No. 337. The Gaseous Explosive Reaction at Constant Pressure—The Reaction Order and Reaction Rate. By F. W. Stevens. 16 pages illustrations, quarto. Washington, Government Printing Office, 1929, price ten cents.

This investigation was carried out at the Bureau of Standards at the request of and with the financial assistance of the Committee.

1. In the case of the gaseous explosive reaction at constant pressure, the data given in this report show that the statistical expression, $\Gamma = [F]_1^{n_1}\,[O_2]_2^{n_2}$, derived from the order of the stoichiometric equation written for complete combustion of a fuel, is proportional to the spatial rate at which an equilibrium is established in the gaseous explosive system, and that this relation is found to hold for high reaction orders where very complex hydrocarbon fuels are involved in the transformation.

2. The above relation, since it is based solely upon the initial and final condition of the transformation, is independent of the microprocesses, whatever these may be, resulting in the final union of the initial active components in the proportions required by the reaction constant K for the temperature and pressure at which the reaction takes place.

3. The data also provide interesting confirmation of the assumption that high-order reaction processes consist of many simultaneous simpler ones each running its course within the reaction zone "according to its own order and mechanism independently of any other reactions occurring at the same time." The probability of the correctness of this assumption is chiefly shown by the fact that the equivalent reaction order of a composite fuel may be determined from the reaction orders of its components, and further, that the velocity constant, k_F of the fuel may also be determined from the velocity constants of those components.

4. The data given in this report all cover the "explosive limits" of the fuels investigated. Incidental to the purpose of the investigation here reported, the "explosive limits" will be found to be expressed for the condition of constant pressure, in the fundamental terms of concentrations (partial pressures) of fuel and oxygen. It may be seen from the results given that a fundamental relation clearly exists between explosive range and the magnitude of $[F]_{max}$ of the fuel.

Report No. 340. Full Scale Wind Tunnel Tests on Several Metal Propellers Having Different Blade Forms. By Fred E. Weick. 13 pages, illustrations, quarto. Washington, Government Printing Office, 1930, price ten cents.

This report gives the full-scale aerodynamic characteristics of five different aluminum alloy propellers having four different blade forms. They were tested on

an open cockpit fuselage with a radial air-cooled engine having conventional cowling, in the Twenty-Foot Propeller Research Tunnel of the National Advisory Committee for Aeronautics, at Langley Field, Va. The results show that (1) the differences in propulsive efficiency due to the differences in blade form were small; (2) the form with the thinnest airfoil sections had the highest efficiency; (3) it is advantageous as regards propulsive efficiency for a propeller operating in front of a body, such as a radial engine, to have its pitch reduced toward the hub.

PUBLICATIONS RECEIVED.

Atoms, Molecules and Quanta, by Arthur Edward Ruark, Ph.D., and Harold Clayton Urey, Ph.D. International Series in Physics. 790 pages, illustrations, 8vo. New York, McGraw-Hill Book Company, Inc., 1930. Price $7.00.

L'Ancienne et la Nouvelle Théorie des Quanta, par Eugene Bloch. Cours de Physique Théorique de la Faculté des Sciences de Paris. 417 pages, illustrations, 8vo. Paris, Librarie Scientifique Hermann et Cie., 1930. Price 90 francs.

Inorganic Qualitative Chemical Analysis, by Allan R. Day, Ph.D. 197 pages, 8vo. Easton, Penna., The Chemical Publishing Company, 1930. Price $2.75.

Wasserkraft Jahrbuch 1828–29 (4 Jahrgang). Herausgeber; Geheimer Baurat Oberbaudirektor Dr. Ing. e. h. K. Dantscher, ord. Professor d. Technischen Hochschule, München; Ingenieur Carl Reindl, München. 487 pages, illustrations, 8vo. München, G. Hirth Verlag, A. G. 1929. Price 22 marks.

Bearing Metals and Bearings, by W. M. Corse. American Chemical Society Monograph Series. 383 pages, illustrations, 8vo. New York, The Chemical Catalog Company, Inc., 1930. Price $7.00.

Electromagnetic Problems in Electrical Engineering. An elementary treatise on the application of the principles of electromagnetism to the theory of the magnetic field and of the mechanical forces in electrical machinery and apparatus by B. Hague, D.Sc. 359 pages, illustrations, 8vo. Oxford University Press, 1929. Price $10.00.

La Théorie de la Relativité et la Mecanique Céleste, par Jean Chazy, Tome II. Collection de physique Mathematique, fascicule III. 258 pages, illustrations, 8vo. Paris, Gauthier-Villars et Cie., 1930. Price 60 francs.

Traité de Polarimetrie, par Georges Bruhat. Préface de A Cotton. 447 pages, illustrations, 8vo. Paris, Revue d'Optique, 1930. Price 65 francs.

La Doctrine Einsteinienne devant les faits périodiques et la refraction lumineuse. Sur l'intelligence au travail par Marcelin Dubroca. 127 pages, illustrations, 8vo. Paris, Gauthier-Villars et Cie., 1930. Price 15 francs.

Year-Book of the International Hydrographic Bureau, Monaco, 1930. 178 pages, 12mo. Monaco, Bureau.

National Advisory Committee for Aeronautics. Technical Notes, No. 332. Injection lags in a common-rail fuel injection system, by A. M. Rothrock. 7 pages, illustrations, quarto. No. 333. Test of an adjustable pitch model propeller at four blade settings by E. P. Lesley. 15 pages, illustrations, quarto. Washington, Committee, 1930.

CURRENT TOPICS.

Leaf Size and Vitamin A Content. LAURA MCLAUGHLIN AND E. S. HABER (*J. Biol. Chem.*, 1929, *84*, 249–256) have used leaves of New Zealand spinach as the sole source of fat-soluble A vitamin in the ration of growing albino rats. Leaves of different size were fed to different animals. The rate of growth of the rats indicated that the potency or content of vitamin A in the leaves depended directly upon their surface area and was inversely proportional to their thickness.

J. S. H.

The International Ozone Conference in Paris, May 15 to May 17, 1929. R. LADENBURG. (*Naturwissenschaften*, July 5, 1929.) In 1920 Fabry and Buisson for the first time measured the quantity of ozone in the total atmosphere above Marseilles by a study of the absorption of the shortest waves of the solar spectrum. In the few years that have elapsed since then the study of the ozone content of the atmosphere has aroused much interest and has drawn to itself a band of investigators. These have already established important relations between the ozone content and other features of climate. Dobson of Oxford has succeeded in getting daily observations of ozone taken over a great part of the earth. It is now believed that the center of gravity of the ozone in the air lies 40 or 50 km. above the earth's surface, that the equivalent depth of the ozone layer reduced to normal gas conditions is 3 mm. and that this varies with season and latitude. Over both hemispheres of the earth there is more ozone present in the spring than in the fall season. Moreover the seasonal change grows greater with the latitude.

There is further a change in the ozone content from day to day which often exceeds that depending on the season. This diurnal variation has a striking connection with changes in the barometric pressure at the earth's surface and at elevations up to 9 km. A "low" goes with high ozone content and a "high" with small content. This relation holds anywhere on the earth where there occurs large changes in the barometer. At the equator where atmospheric pressure varies but little there is correspondingly but little difference from day to day in the ozone content. This absence of change seems to show that an increase or decrease in the number of spots on the sun is without effect upon the quantity of ozone at least at the equator.

Ozone strongly absorbs wave-lengths from 2200 to 3200 Å. In fact it absorbs almost completely the shorter lengths of this interval, even when it is in quite small depth. It is therefore ozone that sets a limit to the short wave-lengths of the solar spectrum and thus determines the physiological effect of the radiation reaching the inhabitants of the earth. Owing to the absorption of solar radiation by the ozone situated about 45 km. above the earth the atmosphere there is considerably warmed. Before the recognition of this Dobson and Lindemann in 1923 had, from a study of the light emitted by meteors, concluded that temperatures at considerable elevations were much higher than in the 10 km. just above the earth. Additional confirmation for this conclusion is furnished by an examination of the anomalous propagation of sound in the atmosphere with its "zones of silence," that can best be explained by assuming the existence of a layer of warm air about 35 or 40 km. above the surface of the earth.

New problems arise from the knowledge of the distribution of ozone and of its importance. The coefficient of absorption of mixtures of ozone and oxygen at atmospheric pressures is measured in the laboratory. Is it legitimate to apply this coefficient to mixtures at pressures of 1 mm. in which ozone has a pressure of only .001 mm.? Has the influence of changes in the amount of vapor in the air been eliminated when variations in ozone content have been presumably observed? Is it possible to deduce the quantity of ozone in the air from a study of the absorption of the yellow-red and infrared portions of the spectrum due to this gas? How is the total quantity of ozone distributed about its center of gravity at 45 km. above the earth? Do the lower layers of the atmosphere really contain no ozone at all as the measurements of Rayleigh and Götz indicate?

The fundamental problem is the origin of the ozone in the air. Wave-lengths shorter than 1800 Å.—and these are present in solar radiation at great heights above the earth—can convert oxygen molecules into ozone. Longer wave-lengths of the ultra-violet on the other hand decompose ozone. Under the influence of these two opposing effects it may be that a state of equilibrium results that represents the permanant ozone layer in the air. The predominant rôle ascribed by this theory to sunlight is, however, not in accord with an observation made in the extreme north of Sweden where the highest ozone content was found after the sun had been low on the horizon for months. May it be that the rays of whatever nature they are, that cause the aurora, are responsible also for the formation of ozone, while the sun's radiation is chiefly effective in decomposing the ozone so formed? Material bearing on this question has, no

doubt been already collected by an expedition sent to Spitzbergen in the summer of 1929. Others of the unsolved problems were distributed among the investigators present.

Great credit is given by Prof. Ladenburg of Berlin to the manner in which Prof. Fabry conducted the conference. It will be remembered that this French physicist was a guest at the centenary of The Franklin Institute in 1924.

G. F. S.

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